Gravitational Contributions to Gauge Green's Functions and Asymptotic Free Power-Law Running of Gauge Coupling

Yong Tang[1,2,*] and Yue-Liang Wu[1,†]

[1] *Kavli Institute for Theoretical Physics China (KITPC)*
State Key Laboratory of Theoretical Physics (SKLTP)
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, 100190, China
[2]*Physics Division, National Center for Theoretical Sciences,*
National Tsing Hua University, Hsinchu

Abstract

We perform an explicit one-loop calculation for the gravitational contributions to the two-, three- and four-point gauge Green's functions with paying attention to the quadratic divergences. It is shown for the first time in the diagrammatic calculation that the Slavnov-Taylor identities are preserved even if the quantum graviton effects are included at one-loop level, such a conclusion is independent of the choice of regularization schemes. We also present a regularization scheme independent calculation based on the gauge condition independent background field framework of Vilkovisky-DeWitt's effective action with focusing on both the quadratic divergence and quartic divergence that is not discussed before. With the harmonic gauge condition, the results computed by using the traditional background field method can consistently be recovered from the Vilkovisky-DeWitt's effective action approach by simply taking a limiting case, and are found to be the same as the ones yielded by the diagrammatic calculation. As a consequence, in all the calculations, the symmetry-preserving and divergent-behavior-preserving loop regularization method can consistently lead to a nontrivial gravitational contribution to the gauge coupling constant with an asymptotic free power-law running at one loop near the Planck scale.

PACS numbers: 11.10.Hi, 04.60.–m

*Electronic address: ytang@itp.ac.cn
†Electronic address: ylwu@itp.ac.cn

I. INTRODUCTION

The classical theory of general relativity has been well verified since its establishment in the beginning of last century. However, the quantum theory of general relativity remains one of the most interesting and frustrating questions. From the standard renormalization analysis, the mass dimension of the coupling $\kappa = \sqrt{32\pi G}$ is negative, which means that general relativity is not a renormalizable theories [1–3]. Since the quantum effects of gravity become important only at the Planck scale $G^{1/2} \approx 10^{19}$GeV, it may suggest that we can treat it an effective field theory [4, 5] at low energy scales.

Gravitational contribution to gauge theories has attracted much attention in recent years. Robinson and Wilczek [6] calculated gravitational corrections to gauge theories in the framework of traditional background-field method, and showed that these corrections can render all gauge theories asymptotically free by changing the gauge couplings to power-law running. This calculation was done in a specific gauge and cut-off regularization. However, it was showed in [7] that the result obtained in [6] was gauge condition dependent, and the gravitational correction to β function at one-loop order was absent in the harmonic gauge. Also, it was found in [8] that, by using gauge-condition independent formalism [9, 10], the gravitational corrections to the β function vanished in dimensional regularization [11]. The above calculations were only involved with gauge two-point Green's function. Later, the authors in [12] performed a diagrammatic calculation of two- and three-point Green's functions in the harmonic gauge by using both cut-off and dimensional regularization schemes, the same conclusion was yielded that quadratic divergences are absent. We should note that all the conclusions are based on one-loop calculations at low energy scale. At or above the Planck scale, the above approximation may break down and new framework for quantum gravity is needed. In this paper, we limit our discussion at one-loop level.

In ref. [13], we have checked all the calculations in the framework of diagrammatic and traditional background field methods, and demonstrated that the results are not only gauge condition dependent but also regularization scheme dependent. A new consistent loop regularization(LORE) method [14] has been applied to carry out the same calculations [13] by using both the diagrammatic and traditional background-field methods. As a consequence, it was found in [13] that there is asymptotic freedom with power-law running in the harmonic gauge condition. Further, various approaches were used to discuss similar issues [15–24].

In this paper, we shall use both diagrammatic approach and Vilkovisky-DeWitt's effective action to calculate in detail the one-loop gravitational corrections to gauge Green's functions and demonstrate explicitly how the gauge invariance is preserved by these corrections. In diagrammatic calculation, two-, three- and four-point gauge Green's functions are computed in a general way. We will show that the Slavnov-Taylor identities are satisfied irrespective of the regularization schemes. Meanwhile, we will also present a calculation by adopting the Vilkovisky-DeWitt's formalism in Einstein-Maxwell system. Both quadratic and quartic divergences can appear in the one loop corrections and thus a proper regularization scheme needs to be applied to handle the quartic divergences to maintain the gauge invariance.

The paper is organized as follows. In Sec. II, we carry out a detailed calculation of one loop gravitational contributions to two-, three- and four-point gauge Green's functions. As a byproduct, the gravitational contribution to the β function of gauge coupling is obtained. In Sec.III, we apply the Vilkovisky-DeWitt's formalism to the Einstein-Maxwell system and show the necessary pieces to calculate the gravitational corrections to the β function of gauge coupling. In Sec. III B, it is shown that the quadratic divergences are presented in a general way, the effects from different regularization schemes are analyzed. Then in Sec. III C, we focus the discussion on the quartic divergence which in general violates gauge invariance and requires proper regularization schemes to handle it. In the end, we shall summarize our results.

II. DIAGRAMMATIC CALCULATION

A. Formalism

The interest of this section is based on the action of Einstein-Yang-Mills theory,

$$\mathrm{S} = \int d^4x\mathcal{L} = \int d^4x\sqrt{-g}\left[\frac{2}{\kappa^2}R - \frac{1}{4}g^{\mu\alpha}g^{\nu\beta}\mathcal{F}^a_{\mu\nu}\mathcal{F}^a_{\alpha\beta}\right], \tag{1}$$

where R is Ricci scalar, $\mathcal{F}^a_{\mu\nu}$ is Yang-Mills fields strength $\mathcal{F}^a_{\mu\nu} = \partial_\mu\mathcal{A}^a_\nu - \partial_\nu\mathcal{A}^a_\mu - ig_0[\mathcal{A}_\mu, \mathcal{A}_\nu]$ and $\kappa = \sqrt{32\pi\mathrm{G}}$. Here and after, repeated indices are summed over in the Einstein summation convention. We expand the metric tensor around a background metric $\bar{g}_{\mu\nu}$ and treat graviton field as quantum fluctuation $h_{\mu\nu}$ propagating on the background space-time determined by $\bar{g}_{\mu\nu}$,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}. \tag{2}$$

Due to the negative mass dimension of coupling constant $\kappa = \sqrt{16\pi \mathrm{G}}$, this theory is not renormalizable.

The above expansion eq. (2) is exact, but the expansions of inverse metric and determinant are approximate by ignoring higher-order terms in realistic calculation. To the second order in κ, we have

$$\begin{aligned} g^{\mu\nu} &= \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}{}_{\alpha} h^{\alpha\nu}, \\ \sqrt{-g} &= \sqrt{-\bar{g}} \left[1 + \frac{1}{2}\kappa h - \frac{1}{4}\kappa^2 \left(h^{\mu\nu} h_{\mu\nu} - \frac{1}{2} h^2 \right) \right]. \end{aligned} \tag{3}$$

The above expansions are two infinite series and the truncation is up to the question considered. We only have to keep terms of order κ or κ^2 when considering the gravitational one-loop correction to pure gauge Green's functions without external graviton line.

For simplicity, we shall consider the case with flat background space-time, $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric, $(1, -1, -1, -1)$. The lagrangian can be arranged to different orders of $h_{\mu\nu}$ or κ. In the gravity part, we work with the de Donder harmonic gauge

$$C^{\mu} = \partial_{\nu} h^{\mu\nu} - \frac{1}{2} \partial^{\mu} h^{\nu}_{\nu} = 0,$$

then, the quadratic terms of $h_{\mu\nu}$ in lagrangian give the graviton's propagator,

$$P_G^{\mu\nu\rho\sigma}(k) = \frac{i}{2k^2} \left[\eta^{\nu\rho}\eta^{\mu\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\nu}\eta^{\rho\sigma} \right]. \tag{4}$$

Graviton shall be labelled as double wiggly line in the Feynman diagrams. For the gauge part, Feynman gauge is used. The interactions of gauge field and gravity field are determined by expanding the second term of the lagrangian (1). And various vertex functions could be derived [12].

B. Renormalization

In Minkowski space-time, the lagrangian for pure Yang-Mills theory is

$$\begin{aligned} \mathcal{L} = &-\frac{1}{4} \mathcal{F}^a_{\mu\nu} \mathcal{F}^{a\mu\nu} = -\frac{1}{4} \left[\partial_{\mu} \mathcal{A}^a_{\nu} - \partial_{\nu} \mathcal{A}^a_{\mu} \right]^2 \\ &- g_0 f_{abc} \left(\partial_{\mu} \mathcal{A}^a_{\nu} \right) \mathcal{A}^{b\mu} \mathcal{A}^{c\nu} - \frac{1}{4} g_0^2 \left(f_{abe} \mathcal{A}^a_{\mu} \mathcal{A}^b_{\nu} \right) \left(f_{cde} \mathcal{A}^{c\mu} \mathcal{A}^{d\nu} \right), \end{aligned}$$

$\mathcal{A}_\mu^a$ and g_0 in the above lagrangian are bare quantities. To remove the divergences appearing in perturbative calculations, both $\mathcal{A}_\mu^a$ and g_0 need to be renormalized,

$$\mathcal{A}_\mu^a = z_2^{1/2} A_\mu^a,\ g_0 = z_g g,$$

z_2 and z_g are referred as field and coupling renormalization constant, respectively. One can also incorporate renormalization into the three and four-point vertices as follows,

$$\begin{aligned}\mathcal{L} =& -\frac{1}{4} z_2 \left[\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right]^2 - z_3 g f_{abc} \left(\partial_\mu A_\nu^a\right) A^{b\mu} A^{c\nu} - \frac{1}{4} z_4 g^2 \left(f_{abe} A_\mu^a A_\nu^b\right) \left(f_{cde} A^{c\mu} A^{d\nu}\right) \\ =& -\frac{1}{4} \left[\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right]^2 - g f_{abc} \left(\partial_\mu A_\nu^a\right) A^{b\mu} A^{c\nu} - \frac{1}{4} g^2 \left(f_{abe} A_\mu^a A_\nu^b\right) \left(f_{cde} A^{c\mu} A^{d\nu}\right) \\ & -\frac{1}{4} \delta_2 \left[\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right]^2 - \delta_3 g f_{abc} \left(\partial_\mu A_\nu^a\right) A^{b\mu} A^{c\nu} - \frac{1}{4} \delta_4 g^2 \left(f_{abe} A_\mu^a A_\nu^b\right) \left(f_{cde} A^{c\mu} A^{d\nu}\right) \end{aligned} \tag{5}$$

with counterterms

$$\delta_2 = z_2 - 1,\ \delta_3 = z_3 - 1,\ \delta_4 = z_4 - 1.$$

The renormalization constants z_3 and z_4 are determined by the divergent part of three- and four-point gauge Green's functions. And both of them have connections with z_g due to gauge invariance, which is well-known as Slavnov-Taylor [28] or Ward identities,

$$z_g = \frac{z_3}{z_2^{3/2}} = \frac{z_4^{1/2}}{z_2}. \tag{6}$$

When fermions and ghosts come in, similar relations exist for their renormalization constants. The running of gauge coupling with renormalization scale μ is described by β function whose definition is

$$\beta(g) \equiv \mu \frac{\partial}{\partial \mu} g.$$

With eq. 6, one can easily have

$$\beta(g) = g\mu \frac{\partial}{\partial \mu} (\frac{3}{2}\delta_2 - \delta_3) = g\mu \frac{\partial}{\partial \mu} (\delta_2 - \frac{1}{2}\delta_4). \tag{7}$$

When we consider the system described by eq. (1), with expanding the metric as eq. (2), many unrenormalizable interactions come in. Even if we only evaluate one loop gravitational corrections to gauge Green's function, operators of higher mass dimension, such as $D_\rho F_{\mu\nu}^a D^\rho F^{a\mu\nu}$, need to be enclosed in the lagrangian. In this paper, we shall limit our discussion in the gravitational contributions to operators appearing in eq. (5). We label the contributions from graviton with a superscript κ,

$$\beta_g^\kappa = g\mu \frac{\partial}{\partial \mu} (\frac{3}{2}\delta_2^\kappa - \delta_3^\kappa) = g\mu \frac{\partial}{\partial \mu} (\delta_2^\kappa - \frac{1}{2}\delta_4^\kappa). \tag{8}$$

Since the interactions of gauge boson and graviton are gauge invariant, the Slavnov-Taylor identities should be preserved automatically. The preservation actually is not trivial at least for two reasons. Firstly, in the realistic calculation, a gauge condition has to be chosen as a gauge fixing condition which generally spoils the gauge invariance, which could potentially destroy Slavnov-Taylor identities. Secondly, at one or higher loop orders, divergences appearing in the loop momentum integral can also break the identities if an improper regularization scheme is used. We shall show explicitly that Slavnov-Taylor identities is maintained and irrespective of the regularization schemes as well.

C. Diagrammatical Calculation

In this subsection, we are going to calculate the quadratic divergences of two, three and four point Green's functions of gauge field. As a byproduct, we can get the β function for the gauge coupling constant. At first, the counterterms in the last line of eq. (5) give vertex functions,

$$\begin{aligned}
\delta\Pi^{\mu\nu}_{ab} &= i\delta_{ab}Q^{\mu\nu}\delta_2,\ \delta T^{\mu\nu\rho}_{abc}(p,q,k) = gf_{abc}V^{\mu\nu\rho}_{pqk}\delta_3,\ \delta T^{\mu\nu\rho\sigma}_{abcd} = -ig^2F^{\mu\nu\rho\sigma}_{abcd}\delta_4, &(9)\\
Q^{\mu\nu} &\equiv q^\mu q^\nu - q^2\eta^{\mu\nu},\ V^{\mu\nu\rho}_{pqk} \equiv \eta^{\mu\nu}(p-q)^\rho + \eta^{\nu\rho}(q-k)^\mu + \eta^{\rho\mu}(k-p)^\nu,\\
F^{\mu\nu\rho\sigma}_{abcd} &\equiv f_{abe}f_{cde}\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) + (b,\nu)\leftrightarrow(c,\rho) + (b,\nu)\leftrightarrow(d,\sigma). &(10)
\end{aligned}$$

In gauge theories without gravity, the counter-terms are logarithmically divergent as the quadratic divergences cancel each other(with proper regularization schemes used) due to gauge symmetry. However, if gravitational corrections are taken into account, divergent behavior becomes different. On dimensional ground, it is known that quadratic divergences can appear and will contribute to the counter-terms defined above, so that they will also lead to the corrections to the β function. In later calculations, we will omit the logarithmic divergences and only focus on the quadratic divergences.

It can be shown that at one-loop level, gravity will contribute to two- and three-point Green's functions as in the following Feynman diagrams, Figs. 1 and 2. For four-point Green's function, two more vertex functions need to be considered, four gauge bosons–one graviton vertex

$$\mathcal{L}_{\rm 4Y1G} = -\frac{1}{4}g^2\kappa f_{abe}f_{cde}A^a_\mu A^b_\nu A^c_\rho A^d_\sigma\left(\frac{1}{2}\eta^{\mu\rho}\eta^{\nu\sigma}h - \eta^{\mu\rho}h^{\nu\sigma} - h^{\mu\rho}\eta^{\nu\sigma}\right), \qquad (11)$$

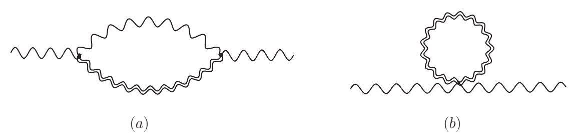

Figure 1: Graviton loop correction to the gauge two-point Green's function.

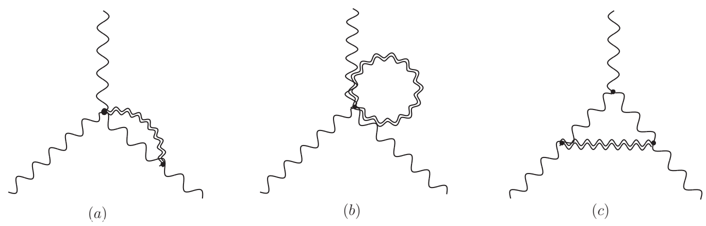

Figure 2: Graviton loop correction to the gauge three-point Green's function.

and four gauge bosons–two gravitons vertex

$$\mathcal{L}_{4\mathrm{Y2G}} = -\frac{1}{4}g^2\kappa^2 f_{abe}f_{cde}A^a_\mu A^b_\nu A^c_\rho A^d_\sigma \left(\frac{1}{8}\eta^{\mu\rho}\eta^{\nu\sigma}\left[h^2 - 2h^{\alpha\beta}h_{\alpha\beta}\right]\right.$$
$$\left. + \eta^{\mu\rho}h^{\nu\beta}h_\beta{}^\sigma + h^{\mu\alpha}h_\alpha{}^\rho\eta^{\nu\sigma} - \frac{1}{2}h\left[h^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\rho}h^{\nu\sigma}\right] + h^{\mu\rho}h^{\nu\sigma}\right). \tag{12}$$

Feynman rules for such vertices can be obtained by standard procedures. The vertex functions are very complicated with many lorentz indices and hundreds of terms, and we evaluate the tensor contraction with *FeynCalc* package [41].

To make the results compact, we introduce the tensor type and scalar type irreducible loop integrals(ILIs) at one-loop level,

$$\mathcal{I}_2(\mathcal{M}^2) \equiv \int \frac{d^4l}{(2\pi)^4}\frac{1}{l^2 - \mathcal{M}^2},\ \mathcal{I}_2^{\mu\nu}(\mathcal{M}^2) \equiv \int \frac{d^4l}{(2\pi)^4}\frac{l^\mu l^\nu}{[l^2 - \mathcal{M}^2]^2}. \tag{13}$$

and for short, use $\mathcal{I}_2$ and $\mathcal{I}_2^{\mu\nu}$ stand for $\mathcal{I}_2(0)$ and $\mathcal{I}_2^{\mu\nu}(0)$, respectively.

After tedious calculation, the two-point function gives

$$\Pi^{\mu\nu}_{ab} \ = \ \kappa^2\delta_{ab}\left[-\frac{1}{2}Q^{\mu\nu}\mathcal{I}_2 + q^\mu q_\rho \mathcal{I}_2^{\nu\rho} + q^\nu q_\rho \mathcal{I}_2^{\mu\rho} - \eta^{\mu\nu}q_\rho q_\sigma \mathcal{I}_2^{\rho\sigma} - q^2\mathcal{I}_2^{\mu\nu}\right],$$

and the three point functions from diagrams (a) and (b) of Fig. 2 are found, when keeping only the quadratically divergent terms, to be

$$T^{\mu\nu\rho}_{abc} = \frac{i}{2}g\kappa^2 f_{abc}\left\{\frac{1}{2}V^{\mu\nu\rho}_{qkp}\mathcal{I}_2 + \left[\eta^{\rho\mu}p_\sigma\mathcal{I}_2^{\nu\sigma} - \eta^{\mu\nu}p_\sigma\mathcal{I}_2^{\rho\sigma} + p^\nu\mathcal{I}_2^{\rho\mu} - p^\rho\mathcal{I}_2^{\mu\nu}\right]\right.$$
$$\left. + \left[\eta^{\mu\nu}q_\sigma\mathcal{I}_2^{\rho\sigma} - \eta^{\nu\rho}q_\sigma\mathcal{I}_2^{\mu\sigma} + q^\rho\mathcal{I}_2^{\mu\nu} - q^\mu\mathcal{I}_2^{\nu\rho}\right] + \left[\eta^{\nu\rho}k_\sigma\mathcal{I}_2^{\mu\sigma} - \eta^{\rho\mu}k_\sigma\mathcal{I}_2^{\nu\sigma} + k^\mu\mathcal{I}_2^{\nu\rho} - k^\nu\mathcal{I}_2^{\rho\mu}\right]\right\}.$$

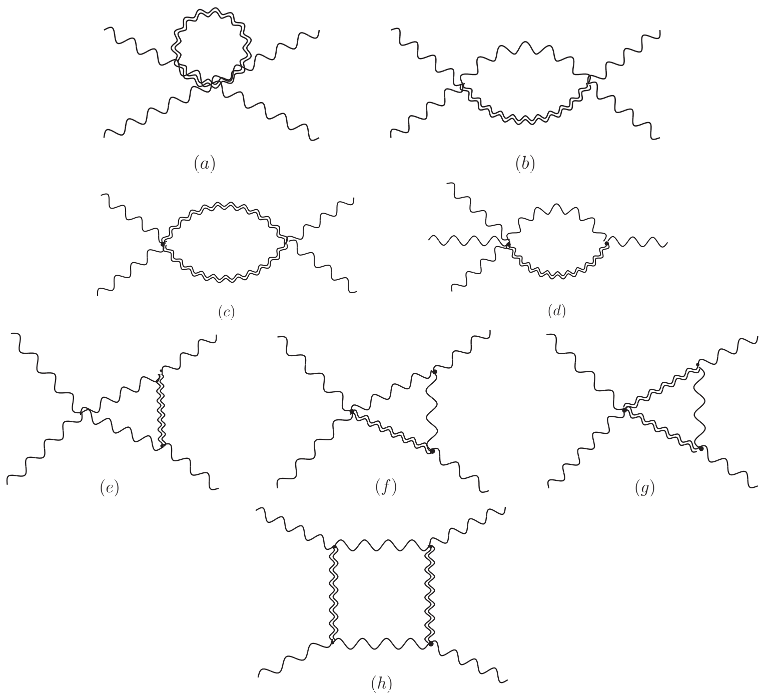

Figure 3: Graviton loop correction to the gauge four-point Green's function.

At one loop level, there are a few Feynman diagrams contributing to the four-point Green's function as shown in Fig. 3. At first sight, the calculation of four point Green's function seems a frustrating task. However, after some analysis on the superficial and real degree of divergence, we can show that only part of these diagrams have quadratic divergence. Superficial degree of divergence is obtained by standard renormalization analysis [29]. We summarize it in the Table I. Where the numbers 2, 1 and 0 stand for quadratic, linear and logarithmic.

Eventually the one-loop gravitational contributions to four point gauge Green's function are found to be

$$\begin{aligned} T^{\mu\nu\rho\sigma}_{abcd} &= g^2\kappa^2 \Big\{ f_{abe}f_{cde}\left[\frac{1}{2}\mathcal{I}_2\left(\eta^{\mu\rho}\eta^{\nu\sigma}-\eta^{\mu\sigma}\eta^{\nu\rho}\right)-\mathcal{I}_2^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}\mathcal{I}_2^{\nu\rho}+\mathcal{I}_2^{\mu\sigma}g^{\nu\rho}-g^{\mu\rho}\mathcal{I}_2^{\nu\sigma}\right] \\ &\quad +(b,\nu)\leftrightarrow(c,\rho)+(b,\nu)\leftrightarrow(d,\sigma)\Big\} \end{aligned}$$

Thus the counterterms δ_2, δ_3 and δ_4 are determined by the quadratically divergent part

Diagrams	Superficial	Real	Order
(a)	2	2	$g^2\kappa^2$
(b)	2	2	$g^2\kappa^2$
(c)	4	0	κ^4
(d)	2	1	$g^2\kappa^2$
(e)	2	0	$g^2\kappa^2$
(f)	2	1	$g^2\kappa^2$
(g)	4	0	κ^4
(h)	4	0	κ^4

Table I: Degree of divergence for each diagram in Fig. 3.

of $\Pi_{ab}^{\mu\nu}$, $T_{abc}^{\mu\nu\rho}$ and $T_{abcd}^{\mu\nu\rho\sigma}$, respectively.

$$\begin{aligned} \Pi_{ab}^{\mu\nu} + i\delta_{ab}Q^{\mu\nu}\delta_2^\kappa &\sim 0, \\ T_{abc}^{\mu\nu\rho} + gf_{abc}V_{pqk}^{\mu\nu\rho}\delta_3^\kappa &\sim 0, \\ T_{abcd}^{\mu\nu\rho\sigma} - ig^2F_{abcd}^{\mu\nu\rho\sigma}\delta_4^\kappa &\sim 0, \end{aligned}$$

from which δ_2^κ,δ_3^κ, and δ_4^κ are determined, respectively. As there are still tensor type quadratic divergences appearing in the above expressions, we shall first reduce them into the scalar type ones. While the subtle can be hidden in such a reducing step, namely it will depend on the regularization schemes which spoil either the symmetry or divergence behavior of original theory, we shall discuss such an issue below in detail.

Regularization: In the cut-off regularization which is known to spoil gauge and translational symmetries, one has

$$\mathcal{I}_2^R = -\frac{i}{16\pi^2}\left[\Lambda^2 - \mu^2\right], \quad \mathcal{I}_{2\mu\nu}^R = \frac{1}{4}g_{\mu\nu}\mathcal{I}_2^R. \tag{14}$$

where the superscript R denote the regularized ones. Putting these formulas into the divergent two-, three- and four-point Green's functions, we straightforwardly get the following results due to cancelations

$$\delta_2^\kappa \sim 0, \quad \delta_3^\kappa \sim 0, \quad \delta_4^\kappa \sim 0.$$

Note that in gauge theories with or without fermions, the relations like eq. (14) will destroy gauge invariance in two-point Green's functions. In the dimensional regularization which

is known to suppress the quadratic divergences, we have $\mathcal{I}_2^R = 0$ and then yield the same results as the ones in the cut-off regularization

$$\delta_2^\kappa \sim 0, \quad \delta_3^\kappa \sim 0, \quad \delta_4^\kappa \sim 0.$$

Note that the vanishes of the above functions in both the cut-off regularization and dimensional regularization have different origins.

We shall adopt the consistent loop regularization(LORE) method[14] which preserves both symmetries and divergence behavior of original theories and has extensively been applied to various calculations with consistent results[30–32]. Recently, the consistency and advantage of the LORE method has further been demonstrated by merging with Bjorken-Drell's analogy between Fynman diagrams and electric circuits and also by explicitly applying to the two-loop regularization and renormalization of ϕ^4 theory[33]. In the LORE method, we have the following consistency condition of gauge invariance for the regularized irreducible loop integrals

$$\mathcal{I}_{2\mu\nu}^R = \frac{1}{2} g_{\mu\nu} \mathcal{I}_2^R \tag{15}$$

with its explicit form given by

$$\mathcal{I}_2^R = \frac{-i}{16\pi^2}\{M_c^2 - \mu_s^2 - \mu_s^2(ln\frac{M_c^2}{\mu_s^2} - \gamma_w)\} \tag{16}$$

where M_c and μ_s play the roles of the ultraviolet and infrared cut-off energy scales. Thus the divergent counterterms are found to be

$$\delta_2^\kappa \sim \frac{i}{2}\kappa^2 \mathcal{I}_2^R, \quad \delta_3^\kappa \sim \frac{i}{2}\kappa^2 \mathcal{I}_2^R, \quad \delta_4^\kappa \sim \frac{i}{2}\kappa^2 \mathcal{I}_2^R. \tag{17}$$

which can be shown to satisfy the Slavnov-Taylor identities eq.(6). In fact, one can easily check that as long as the consistency condition of gauge invariance eq.(15) is imposed, the Slavnov-Taylor identities eq.(6) are preserved no matter which regularization scheme is used.

The β function: Putting the leading quadratically divergent parts of δ_2^κ and δ_3^κ(or δ_4^κ) into eq.(8), we obtain the gravitational corrections to the gauge β function

$$\beta_g^\kappa = -g\kappa^2 \frac{\mu^2}{32\pi^2} \tag{18}$$

which shows that there are gravitational quadratic corrections to the gauge β function when the LORE method is adopted to evaluate the quadratic divergent integrals, which is different from the results yielded by using the cut-off and dimensional regularization schemes.

Note that for an abelian gauge theory, there are no counterterms for δ_3^κ and δ_4^κ. Thus in the abelian gauge case, the renormalization constant of gauge coupling z_g is related to that of gauge field z_2 with $z_g z_2^{1/2} = 1$, the corresponding β-function correction is given via $\beta_g^\kappa = \frac{1}{2} g\mu \frac{\partial}{\partial\mu}\delta_2^\kappa$, which leads to the same result as eq. (18). Therefore, the gravitational correction to the running of gauge coupling is universal for all gauge theories.

III. VILKOVISKY-DEWITT'S BACKGROUND FIELD METHOD

In this section, we shall apply Vilkovisky-DeWitt's background field method to Einstein-Maxwell system other than Einstein-Yang-Mills systerm, for simplicity. This section is partly overlapped with [25] about the action expansion and with [21, 26, 27] about quadratic divergences, but we shall present a complete calculation in a regularization independent way and pay attention to the quartic divergence which has not been discussed before. Details of the calculation are given in the appendix.

For a comparison with the results obtained in [25], we shall use the same notation. Especially, the DeWitt's condensed index notation is used throughout below, except for places where an explicit calculation is given. In the appendix, a short review of the effective action is given. In a general gauge condition, the resulting Vilkovisky-DeWitt's effective action is given by

$$\Gamma[\bar{\varphi}] = S[\bar{\varphi}] - \ln\det Q^\alpha{}_\beta + \frac{1}{2}\ln\det\left(\nabla^i\nabla_j S[\bar{\varphi}] + \frac{1}{2\Omega}\chi_\alpha{}^{,i}\chi^\alpha{}_{,j}\right). \tag{19}$$

where χ_α is the gauge condition, $Q^\alpha{}_\beta$ is the Faddeev-Popov factor, $\bar{\varphi}$ is the background field, and $\nabla_i\nabla_j S[\bar{\varphi}] = S_{,ij}[\bar{\varphi}] - \Gamma^k_{ij} S_{,k}[\bar{\varphi}]$ with Γ^k_{ij} being given and explained in the appendix.

A. Action Expansion

We expand the fields, $\varphi^i = (g_{\mu\nu}, A_\mu)$, at the flat background-fields, $\bar{\varphi}^i = (\delta_{\mu\nu}, \bar{A}_\mu)$,

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu}; \quad A_\mu = \bar{A}_\mu + a_\mu. \tag{20}$$

and label $\eta^i = (h_{\mu\nu}, a_\nu)$ as the graviton and photon fields. Due to the complicated $\bar{\Gamma}^k_{ij}$, it is much simpler to work in Landau-DeWitt gauge ($\omega = 1$), $K_{\alpha i}[\bar{\varphi}]\eta^i = 0$. Explicitly, we have

$$\begin{aligned} \chi_\lambda &= \frac{2}{\kappa}(\partial^\mu h_{\mu\lambda} - \frac{1}{2}\partial_\lambda h) + \omega(\bar{A}_\lambda \partial^\mu a_\mu + a^\mu \bar{F}_{\mu\lambda}) \\ \chi &= -\partial^\mu a_\mu. \end{aligned}$$

$K^i_\alpha[\bar{\varphi}]$ is the generator of gauge transformations. Details can be referred to the appendix. In this gauge, the resulting Vilkovisky-DeWitt's effective action is given by

$$\begin{aligned}\Gamma[\bar{\varphi}] \;=\; & S[\bar{\varphi}] - \ln\det Q_{\alpha\beta}[\bar{\varphi}] \\ & + \frac{1}{2}\lim_{\Omega\to 0}\ln\det\left(\nabla^i\nabla_j S[\bar{\varphi}] + \frac{1}{2\Omega}K^i_\alpha[\bar{\varphi}]K^\alpha_j[\bar{\varphi}]\right)\end{aligned}$$

with $\nabla_i\nabla_j S[\bar{\varphi}] = S_{,ij}[\bar{\varphi}] - \Gamma^k_{ij}S_{,k}[\bar{\varphi}]$, and the connection Γ^k_{ij} is determined by $g_{ij}[\varphi]$ which is the metric on the field space. In the resulting effective action, a parameter, v, is introduced for the connection term [25],

$$S_q = \frac{1}{2}\eta^i\left(S_{,ij} - v\Gamma^k_{ij}S_{,k} + \frac{1}{2\Omega}K_{\alpha i}K^\alpha_j\right)\eta^j. \tag{21}$$

Note that both ω and v are not real gauge condition parameters, and their values are actually fixed in Landau-DeWitt gauge, $\omega = 1$, $v = 1$. They are introduced here just for an advantage of comparing with the traditional background field method in harmonic gauge by simply taking $\omega = 0$, $v = 0$. In principle, the Vilkovisky-DeWitt formalism is applicable in any gauge condition as it has been verified to be gauge condition independent[37, 38, 40]. While in a practical calculation, such a formalism becomes much simpler in Landau-DeWitt gauge at one loop. Therefore, we will impose eventually the Landau-DeWitt gauge condition: $\omega = 1$, $v = 1$, $\xi \to 0$ and $\zeta \to 0$ to obtain a gauge condition independent result as guaranteed by the Vilkovisky-DeWitt formalism. Meanwhile, by taking $\omega = 0$, $v = 0$, and $\xi = 1/\kappa^2$, $\zeta = 1/2$, we can straightforwardly read out the result in the traditional background field method in harmonic gauge.

By appropriately arranging all the terms in the expanded action, we can express S_q in eq.(21) as follows

$$S_q = S_0 + S_1 + S_2, \tag{22}$$

which is found to be consistent with that in [25] where terms from $\bar{A}_\lambda\partial^\mu a_\mu$ in gauge condition are neglected for evaluation of logarithmic divergences. Later we will show that such terms could lead to quartic divergences and violate gauge invariance.

The free part can be written as

$$\begin{aligned}S_0 \;=\; & \int d^4x\left[-\frac{1}{2}h^{\mu\nu}\Box h_{\mu\nu} + \frac{1}{4}h\Box h + \left(\frac{1}{\kappa^2\xi} - 1\right)\left(\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h\right)^2 - \Lambda\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right)\right. \\ & \left. + \frac{1}{2}a_\mu\left(-\delta^{\mu\nu}\Box + \partial^\mu\partial^\nu\right)a_\nu + \frac{1}{4\zeta}\left(\partial^\mu a_\mu\right)^2 - \frac{v}{2}\Lambda\delta^{\mu\nu}a_\mu a_\nu\right]\end{aligned} \tag{23}$$

with Λ the cosmological constant. The interaction terms with linear on graviton $h_{\mu\nu}$ or gauge field a_μ have the following form

$$
\begin{aligned}
S_1 &= \frac{\kappa}{2}\int d^4x\left(\bar{F}^{\mu\nu}h\partial_\mu a_\nu - 2\bar{F}_\alpha{}^\nu h^{\mu\alpha}\partial_\mu a_\nu + 2\bar{F}_\alpha{}^\nu h^{\mu\alpha}\partial_\nu a_\mu\right) \\
&\quad -\frac{\kappa v}{4}\int d^4x\left(\delta_\sigma^\lambda\delta^{\mu\nu} - 2\delta_\sigma^{(\mu}\delta^{\nu)\lambda}\right)\partial_\tau\bar{F}^{\sigma\tau}h_{\mu\nu}a_\lambda \\
&\quad +\frac{\omega}{\kappa\xi}\int d^4x\left(\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h\right)\left(\bar{A}^\nu\partial^\lambda a_\lambda + a^\lambda\bar{F}_\lambda{}^\nu\right) \\
&= \int d^4x\left[C_{11}^{\alpha\beta\mu\nu}h_{\alpha\beta}\partial_\mu a_\nu + C_{12}^{\alpha\beta\mu}h_{\alpha\beta}a_\mu + \frac{\omega}{\kappa\xi}C_{13}^{\nu\alpha\beta}\partial_\nu h_{\alpha\beta}\partial^\mu a_\mu\right] = S_{11} + S_{12} + S_{13}
\end{aligned}
$$

and the interaction terms with quadratic on graviton $h_{\mu\nu}$ or gauge field a_μ are given by

$$
\begin{aligned}
S_2 &= \frac{\kappa^2}{4}\int d^4x\bar{F}_{\mu\nu}\bar{F}_{\alpha\beta}\left(2\delta^{\mu\alpha}h_\lambda^\nu h^{\lambda\beta} + h^{\mu\alpha}h^{\nu\beta} - \delta^{\mu\alpha}hh^{\nu\beta}\right) - \frac{\kappa^2}{16}\int d^4x\bar{F}^2\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right) \\
&\quad +\frac{\kappa^2 v}{4}\int d^4x\left(\frac{1}{2}\bar{F}^\lambda{}_\gamma\bar{F}^{\sigma\gamma}\delta^{\mu\nu} - \bar{F}^\mu{}_\gamma\bar{F}^{\sigma\gamma}\delta^{\nu\lambda} + \left[\frac{1}{4}\delta^{\mu\lambda}\delta^{\sigma\nu} - \frac{1}{8}\delta^{\mu\nu}\delta^{\lambda\sigma}\right]\bar{F}^2\right)h_{\mu\nu}h_{\lambda\sigma} \\
&\quad -\int d^4x\left[\frac{\kappa^2 v}{4}\left(\frac{1}{8}\delta^{\mu\nu}\bar{F}^2 - \frac{1}{2}\bar{F}^\mu{}_\gamma\bar{F}^{\nu\gamma}\right) - \frac{\omega^2}{4\xi}\bar{F}^\mu{}_\gamma\bar{F}^{\nu\gamma}\right]a_\mu a_\nu + \frac{\omega^2}{4\xi}\int d^4x\bar{A}_\lambda\bar{A}^\lambda\partial^\mu a_\mu\partial^\nu a_\nu \\
&= \int d^4x\left[C_{21}^{\alpha\beta\mu\nu}h_{\alpha\beta}h_{\mu\nu} + C_{22}^{\mu\nu}a_\mu a_\nu + \frac{\omega^2}{4\xi}C_{23}\partial^\mu a_\mu\partial^\nu a_\nu\right] = S_{21} + S_{22} + S_{23}.
\end{aligned}
\tag{24}
$$

The tensor coefficients, $C_{11}^{\alpha\beta\mu\nu}$, $C_{12}^{\alpha\beta\mu}$, $C_{13}^{\nu\alpha\beta}$, $C_{21}^{\alpha\beta\mu\nu}$, $C_{22}^{\mu\nu}$ and C_{23} are functions of $\bar{A}_\mu$ and $\delta_{\mu\nu}$, and they can be read out directly. The graviton and photon propagators are determined by S_0. And the terms in S_1 and S_2 will be treated as interactions between background fields and quantum fields. Note that S_{13} and S_{23} are proportional to ω, and they result from the $\bar{A}_\lambda\partial^\mu a_\mu$ term in the Landau-DeWitt gauge condition for graviton given in eq.(E12). We shall show that these two terms do not contribute to the effective action in the present choice of the gauge condition.

We can write the photon propagator in momentum space as [25]

$$
\langle a_\mu(x)a_\nu(x')\rangle = G_{\mu\nu}(x,x') = \int\frac{d^4p}{(2\pi)^4}e^{ip\cdot(x-x')}G_{\mu\nu}(p),
$$

and the graviton propagator as

$$
\langle h_{\rho\sigma}(x)h_{\lambda\tau}(x')\rangle = G_{\rho\sigma\lambda\tau}(x,x') = \int\frac{d^4p}{(2\pi)^4}e^{ip\cdot(x-x')}G_{\rho\sigma\lambda\tau}(p).
$$

Using the explicit result for S_0, we have

$$
G_{\mu\nu}(p) = \frac{\delta_{\mu\nu}}{p^2 - v\Lambda} + (2\zeta - 1)\frac{p_\mu p_\nu}{(p^2 - v\Lambda)(p^2 - 2\zeta v\Lambda)}
\tag{25}
$$

and

$$G_{\rho\sigma\lambda\tau}(p) = \frac{\delta_{\rho(\lambda}\delta_{\tau)\sigma} - \frac{1}{2}\delta_{\rho\sigma}\delta_{\lambda\tau}}{(p^2 - 2\Lambda)} + (\kappa^2\xi - 1)\frac{\delta_{\rho(\lambda}p_{\tau)}p_\sigma + \delta_{\sigma(\lambda}p_{\tau)}p_\rho}{(p^2 - 2\Lambda)\,(p^2 - 2\kappa^2\xi\Lambda)} \tag{26}$$

Now we are ready to check that S_{13} and S_{23} give vanishing quadratic divergences. In the harmonic gauge, $\omega = 0$, it becomes manifest. In the Landau-DeWitt gauge, $\omega = 1$ and $\zeta = 0$, the gauge propagator is

$$G_{\mu\nu}(p) = \frac{1}{p^2}\left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right]$$

Since the interactions in S_{13} and S_{23} have a factor of $\partial^\mu a_\mu$, then $p^\mu G_{\mu\nu}$ and $p^\mu p^\nu G_{\mu\nu}$ will appear in the tensor contraction, which give vanishing contributions. However, in a general gauge, we will encounter quartic divergent integrals like $\bar{A}_\mu \bar{A}^\mu \int \frac{d^4p}{(2\pi)^2} 1$, such a quartically divergent term has to be well regularized, otherwise it will violate the $U(1)$ gauge symmetry due to its contribution to the gauge boson mass. We shall discuss it further next section.

A similar analysis can be made for the effective action of ghost part. The free part can be written down as

$$S_{GH0} = \int d^4x \left[-\frac{2}{\kappa^2}\bar{c}^\lambda \Box c_\lambda - \bar{c}\Box c\right] \tag{27}$$

The interaction term with linear on gravity ghost or gauge ghost is given by

$$S_{GH1} \;=\; \int d^4x \Big\{\omega \bar{c}^\lambda \bar{F}_{\mu\lambda} c^{,\mu} + \omega \bar{c}^\lambda \bar{A}_\lambda \Box c + \left[\bar{c}\bar{A}_{\nu,\mu} c^{\nu,\mu} - \bar{c}^{,\mu}\bar{A}_{\mu,\nu} c^\nu + \bar{c}\bar{A}_\nu \Box c^\nu\right]\Big\} \tag{28}$$

and the interaction term with quadratic terms on gravity ghost has the form

$$S_{GH2} = \omega \int d^nx \Big\{\bar{c}^\lambda \bar{F}_{\lambda\mu}\left[\bar{A}^\mu{}_{,\nu} c^\nu + \bar{A}_\nu c^{\nu,\mu}\right] - \left[\bar{c}^\lambda \bar{A}_\lambda \bar{A}_\rho \Box c^\rho + \bar{c}^\lambda \bar{A}_\lambda \bar{A}_{\nu,\rho} c^{\rho,\nu}\right]\Big\}. \tag{29}$$

From the free part S_{GH0}, the ghosts' propagators can easily be read off

$$\begin{aligned}
\langle c_\mu(x)\bar{c}_\nu(x')\rangle &= \Delta_{\mu\nu}(x,x') = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-x')}\Delta_{\mu\nu}(p), \\
\langle c_\mu(x) c_\nu(x')\rangle &= \langle \bar{c}_\mu(x)\bar{c}_\nu(x')\rangle = 0 \\
\langle c(x)\bar{c}(x')\rangle &= \Delta(x,x') = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-x')}\Delta(p), \\
\langle c(x) c(x')\rangle &= \langle \bar{c}(x)\bar{c}(x')\rangle = 0
\end{aligned}$$

where the propagators in the momentum space are given by

$$\Delta_{\mu\nu}(p) = \frac{\kappa^2}{2}\delta_{\mu\nu}\frac{1}{p^2}, \quad \Delta(p) = \frac{1}{p^2}$$

B. One-Loop quadratically divergent contribution

In this subsection, we shall present our results for the quadratically divergent contributions. As a consistent check, we have reproduced the results for the logarithmic divergent contributions to the β function when the cosmological constant is included[25]. Here we shall not repeat the similar analysis and only carry out the calculation for the leading quadratic divergences which are encountered in corrections to $\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. The contributions from the gravity-gauge interactions to the effective action can be written as

$$\Gamma_G = \langle S_2\rangle - \frac{1}{2}\langle S_1^2\rangle, \quad \langle S_2\rangle = \langle S_{21}\rangle + \langle S_{22}\rangle \tag{30}$$

with the explicit forms given by

$$\langle S_{21}\rangle = \int d^4x C_{21}^{\alpha\beta\mu\nu} G_{\alpha\beta\mu\nu}(x,x) \tag{31}$$

$$\langle S_{22}\rangle = \int d^4x C_{22}^{\mu\nu} G_{\mu\nu}(x,x) \tag{32}$$

$$\langle S_1^2\rangle = \int d^4x \int d^4x' C_{11}^{\alpha\beta\mu\nu} C_{11}^{\rho\sigma\lambda\tau} G_{\alpha\beta\rho\sigma}(x,x')\partial_\mu\partial'_\lambda G_{\nu\tau}(x,x') \tag{33}$$

where we have neglected the corrections in the $\langle S_1^2\rangle$ from the term $C_{12}^{\alpha\beta\mu}$ which contributes to high order operators. S_{13} and S_{23} give vanishing quadratic divergences as we have explained in the previous section. Thus we can expand the gauge and graviton propagators into

$$\begin{aligned} G_{\mu\nu}(p) &= \frac{\delta_{\mu\nu}}{p^2 - v\Lambda} + (2\zeta - 1)\frac{p_\mu p_\nu}{(p^2 - v\Lambda)(p^2 - 2\zeta v\Lambda)} \\ &= \frac{\delta_{\mu\nu}}{p^2} + (2\zeta - 1)\frac{p_\mu p_\nu}{p^4} + \mathcal{O}(\Lambda) \end{aligned} \tag{34}$$

and

$$\begin{aligned} G_{\rho\sigma\lambda\tau}(p) &= \frac{\delta_{\rho(\lambda}\delta_{\tau)\sigma} - \frac{1}{2}\delta_{\rho\sigma}\delta_{\lambda\tau}}{(p^2 - 2\Lambda)} + (\kappa^2\xi - 1)\frac{\delta_{\rho(\lambda}p_{\tau)}p_\sigma + \delta_{\sigma(\lambda}p_{\tau)}p_\rho}{(p^2 - 2\Lambda)\,(p^2 - 2\kappa^2\xi\Lambda)} \\ &= \frac{\delta_{\rho(\lambda}\delta_{\tau)\sigma} - \frac{1}{2}\delta_{\rho\sigma}\delta_{\lambda\tau}}{p^2} + (\kappa^2\xi - 1)\frac{\delta_{\rho(\lambda}p_{\tau)}p_\sigma + \delta_{\sigma(\lambda}p_{\tau)}p_\rho}{p^4} + \mathcal{O}(\Lambda) \end{aligned} \tag{35}$$

Since the cosmological constant Λ is of mass-dimension two, the corrections arising from $\mathcal{O}(\Lambda)$ are only logarithmically divergent, we shall not consider its effects below. When the calculation involves propagators $G_{\mu\nu}(p-q)$ and $G_{\rho\sigma\lambda\tau}(p-q)$ which depend on the external momentum q, we will treat respectively $G_{\mu\nu}(p-q)$ as $G_{\mu\nu}(p)$ and $G_{\rho\sigma\lambda\tau}(p-q)$ as $G_{\rho\sigma\lambda\tau}(p)$, since the q-dependent contributions can be regarded as the higher order terms,

like $\partial^\mu F_{\alpha\beta}\partial_\mu F^{\alpha\beta}$. With this consideration and approximation, all the remaining leading-contributions will only involve with the following quadratically divergent tensor- and scalar-type loop integrals

$$\mathcal{I}_{2\mu\nu} = \int d^4p \frac{p_\mu p_\nu}{p^4}; \quad \mathcal{I}_2 = \int d^4p \frac{1}{p^2}. \tag{36}$$

In general, one needs a consistent regularization to make the quadratically divergent integrals well-defined. Without involving the details of regularization schemes, one can always relate the regularized tensor-type integral with the regularized scalar-type integral via the general Lorentz structure as follows

$$\mathcal{I}^R_{2\mu\nu} = a_2 \delta_{\mu\nu} \mathcal{I}^R_2. \tag{37}$$

where the superscript R denotes the regularized integral. Here a_2 may be different in different regularization schemes. However, by explicitly calculating one loop diagrams of gauge theories, it has been shown [14] that a consistency condition with

$$a_2 = \frac{1}{2} \tag{38}$$

is required to preserve gauge invariance for $\mathcal{I}^R_2 \neq 0$. In the LORE method[14], the condition Eq. (38) is satisfied, while the naive cut-off regularization does not lead to the condition Eq. (38) as it results to $a_2 = \frac{1}{4}$. The dimensional regularization is known to suppress the quadratic divergence and gives $\mathcal{I}^R_2 = 0$, which leads no quadratic divergence.

With the above general relation, the quadratically divergent parts of effective action, without including ghost's contribution at one-loop level, are found to be

$$\begin{aligned} \langle S_2 \rangle &= \kappa^2 (C_{21} + C_{22}) \mathcal{I}^R_2 \frac{1}{4} \int d^4x \bar{F}^2 \\ \langle S_1^2 \rangle &= \kappa^2 C_{11} \mathcal{I}^R_2 \frac{1}{4} \int d^4x \bar{F}^2 \end{aligned} \tag{39}$$

where the numerical factors are explicitly given by

$$\begin{aligned} C_{21} &= \frac{1}{2}\Big([v(1-4a_2) + 6a_2](\kappa^2\xi - 1) + 3 \Big) \\ C_{22} &= \frac{v}{8}(4a_2 - 1)(2\zeta - 1) + \frac{\omega^2}{\kappa^2\xi}\Big[(2\zeta - 1)a_2 + 1 \Big] \\ C_{11} &= \frac{2\omega^2}{\kappa^2\xi}([2\zeta - 1]a_2 + 1) + 2\kappa^2\xi(1 - a_2) + 6a_2 - 4\omega(1 - a_2) \end{aligned} \tag{40}$$

Thus the effective action with the total gravitational field contributions is found at the

one-loop order to be

$$\Gamma_G = \langle S_2\rangle - \frac{1}{2}\langle S_1^2\rangle = \kappa^2 C_G \mathcal{I}_2^R \frac{1}{4}\int d^4x \bar{F}^2 \tag{41}$$

$$\begin{aligned} C_G &= \frac{(4a_2-1)}{8}\Big(v\left[(2\zeta-1)-4(\kappa^2\xi-1)\right] \\ &+ 8(\kappa^2\xi-1)-16\omega-4\Big)+6\omega a_2 \end{aligned} \tag{42}$$

Thanks the cancelation of $1/\xi$ terms in $\langle S_2\rangle$ and $\langle S_1^2\rangle$, otherwise it would be inconsistent when going back to the Landau-DeWitt gauge $\xi\to 0$. The ghost's contribution to the effective action at the one-loop order can be written as

$$\Gamma_{GH} = \langle S_{GH2}\rangle - \frac{1}{2}\langle S_{GH1}^2\rangle \tag{43}$$

An additional sign has to be taken care for a ghost loop in the calculation, the quadratically divergent contributions are found to be

$$\langle S_{GH2}\rangle = -\kappa^2\omega\mathcal{I}_2^R\frac{1}{4}\int d^4x\bar{F}^2 \ ; \quad \left\langle S_{GH1}^2\right\rangle = 0 \tag{44}$$

which is independent of a_2 in Eq. (37).

Thus the total quadratically divergent one-loop gravitational contribution to the effective action has the following gauge invariant form

$$\Gamma = \frac{1}{4}\int d^4x\bar{F}^2 + \kappa^2 C\mathcal{I}_2^R\frac{1}{4}\int d^4x\bar{F}^2 \tag{45}$$

where the constant C is given by

$$\begin{aligned} C &= C_G-\omega = \frac{4a_2-1}{8}\Big(v\left[(2\zeta-1)-4(\kappa^2\xi-1)\right] \\ &+ 8(\kappa^2\xi-1)-16\omega-4\Big)+\omega(-1+6a_2) \end{aligned} \tag{46}$$

Thus the corresponding counter-term is needed to renormalize the gauge field and gauge coupling constant. The renormalized gauge action is given by

$$\Gamma = \frac{1}{4}(1+\delta_2)\int d^4x\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} \tag{47}$$

where δ_2 is determined via the cancelation of the quadratic divergence $\delta_2+\kappa^2 C\mathcal{I}_2^R\simeq 0$, namely

$$\delta_2 \simeq -\kappa^2 C\mathcal{I}_2^R \tag{48}$$

as the charge renormalization constant z_e is connected to the gauge field renormalization constant $z_2 = 1+\delta_2$ via the identity $z_e z_2^{1/2} = 1$, the gravitational correction to the β function is defined to be

$$\beta_e^\kappa = \mu\frac{\partial}{\partial\mu}e = \mu\frac{\partial}{\partial\mu}z_e^{-1}e^0 = \frac{1}{2}e^0\mu\frac{\partial}{\partial\mu}\delta_2 \tag{49}$$

from which we can obtain the gravitational corrections to the β function

$$\beta_e^\kappa = \frac{\mu^2}{16\pi^2}e\kappa^2 C, \tag{50}$$

where e is the electric charge. Such a result indicates that there do exist quadratically divergent gravitational contributions to the gauge coupling constant for $C \neq 0$. Let us now impose the Landau-DeWitt gauge condition $v = 1,\ \omega = 1,\ \zeta = 0,\ \xi = 0$, and take the gauge invariance consistency condition $a_2 = 1/2$, which leads to a nonzero value for the constant $C = 6a_2 - 1 - 25(a_2/2 - 1/8) = -9/8$ and results in a negative β function

$$\beta_e^\kappa = -\frac{9\mu^2}{128\pi^2}e\kappa^2 \tag{51}$$

We would like to emphasize that the above result is gauge condition independent ensured by the Vilkovisky-DeWitt formalism, and is also independent of any specific regularization schemes as long as the regularization schemes preserve gauge symmetry and divergent behavior. We then arrive at the statement that gravity does provide power-law contributions to the gauge coupling constant and tends to make gauge coupling asymptotically free.

We are also in the position to make comments on the regularization scheme dependence. In the dimension regularization, one has $\mathcal{I}_2^R = 0$, so that $\delta_2 = 0$ and $\beta_e^\kappa = 0$, it is then manifest that there is no quadratically divergent gravitational contributions in any case based on the dimensional regularization. In the cut-off regularization, one has $a_2 = 1/4$ and $C = 1/2$, namely $\beta_e^\kappa = \mu^2/(32\pi^2)e\kappa^2$ which leads to no asymptotic freedom.

As a consistent check, let us revisit the traditional background field method in the harmonic gauge, which is recovered by simply taking $v = 0,\ \omega = 0,\ \zeta = 1/2,\ \xi = 1/\kappa^2$ in the above Vilkovisky-DeWitt formalism. As a consequence, it leads to

$$C = 1/2 - 2a_2 \tag{52}$$

which becomes manifest that in the cut-off regularization, one has $a_2 = 1/4$, $C = 0$ and $\beta_e^\kappa = 0$, which confirms the previous results given in [7, 12, 13]. In the LORE method $a_2 = 1/2$,

we have $C = -1/2$ and $\beta_e^\kappa = -\mu^2/(32\pi^2)e\kappa^2$ which confirms our previous result given in Eq. (18) and ref. [13]. Namely, the quadratically divergent gravitational contribution to the gauge coupling constant is asymptotic free in the traditional background field or equivalently in the diagrammatic method with the harmonic gauge.

C. Quartic divergences

In this section, we shall restrict ourselves to the quartic divergences that may appear in the calculation. The quartic divergences have a form of

$$\bar{A}^\mu \bar{A}_\mu \int \frac{d^4p}{(2\pi)^4} 1 = \bar{A}^\mu \bar{A}_\mu \mathcal{I}_4. \tag{53}$$

such a term will violate the $U(1)$ gauge symmetry without adopting a proper regularization scheme to handle it.

In the gravity-gauge sector, it is easy to check that quartic divergences only show in the contributions from $\langle S_{13}^2 \rangle$ and $\langle S_{23} \rangle$. Both contributions are proportional to ω^2/ξ,

$$\frac{1}{2}\left\langle S_{13}^2 \right\rangle_4 = \frac{\omega^2}{2\kappa^2\xi^2} \int d^4x \int d^4x' C_{13}^{\mu\alpha\beta} C_{13}^{\nu\rho\sigma} \partial_\mu \partial'_\nu G_{\alpha\beta\rho\sigma}(x,x') \partial^\gamma \partial'^\tau G_{\gamma\tau}(x,x') \tag{54}$$

with $C_{13}^{\mu\alpha\beta} = \delta^{\mu(\alpha}\bar{A}^{\beta)} - \frac{1}{2}\delta^{\alpha\beta}\bar{A}^\mu$ and $\partial^\gamma \partial'^\tau G_{\gamma\tau}(x,x') = 2\zeta\delta(x,x')$. Here the subscript 4 indicates that we are dealing with quartic divergence. We can show that only the terms proportional to $\kappa^2\xi$ in the gravity propagator contribute to the quartic divergent term,

$$\frac{1}{2}\left\langle S_{13}^2 \right\rangle_4 = \frac{\omega^2}{4\xi} 2\zeta \int d^4x \bar{A}^\mu \bar{A}_\mu \mathcal{I}_4 \tag{55}$$

and similarly we have

$$\begin{aligned} \langle S_{23} \rangle_4 &= \frac{\omega^2}{4\xi} \int d^4x C_{23} \partial^\gamma \partial^\tau G_{\gamma\tau}(x,x) \\ &= \frac{\omega^2}{4\xi} 2\zeta \int d^4x \bar{A}^\mu \bar{A}_\mu \mathcal{I}_4 \end{aligned} \tag{56}$$

Luckily, the cancelation occurs that $\langle S_{23} \rangle_4 - \frac{1}{2}\left\langle S_{13}^2 \right\rangle_4 = 0$. If they could not cancel each other, we would get inconsistent result in the limit of $\xi \to 0$. Again, we emphasize that we only confine our discussion here to quartic divergence. In a general gauge for arbitrary ω, the connection term S_T in Eq.(B28) should be included as well. Since S_T involves $S_{,i}$, Eq. (E2) has only $\bar{F}_{\mu\nu}$ or $\partial_\mu \bar{F}^{\mu\nu}$ in the absence of cosmological constant Λ, there will be no quartically divergent correction to $\bar{A}_\mu \bar{A}^\mu$ from S_T.

However, in the ghost sector there is no such a cancelation. The second term $\omega\bar{c}^{\lambda}\bar{A}_{\lambda}\Box c$ in eq.(28), and the terms in the second bracket of eq.(29), $-\omega\left[\bar{c}^{\lambda}\bar{A}_{\lambda}\bar{A}_{\rho}\Box c^{\rho}+\bar{c}^{\lambda}\bar{A}_{\lambda}\bar{A}_{\nu,\rho}c^{\rho,\nu}\right]$, originate from the interaction term $\omega\bar{A}_{\lambda}\partial^{\mu}a_{\mu}$ in the Landau-DeWitt gauge condition for graviton in eq.(E12). These two terms will give a non-zero quartic divergence.

$$\begin{aligned}\frac{1}{2}\left\langle S_{GH1}^{2}\right\rangle_{4} &= \frac{1}{2}\omega\left\langle\int d^{4}x\int d^{4}x'\bar{c}^{\lambda}\bar{A}_{\lambda}\Box c\bar{c}\bar{A}_{\nu}\Box c^{\nu}\right\rangle\\ &= \frac{\kappa^{2}}{4}\omega\int d^{4}x\bar{A}^{\mu}\bar{A}_{\mu}\mathcal{I}_{4}\end{aligned} \tag{57}$$

and

$$\begin{aligned}\left\langle S_{GH2}\right\rangle_{4} &= -\omega\left\langle\int d^{4}x\bar{c}^{\lambda}\bar{A}_{\lambda}\bar{A}_{\nu}\Box c^{\nu}\right\rangle\\ &= -\frac{\kappa^{2}}{2}\omega\int d^{4}x\bar{A}^{\mu}\bar{A}_{\mu}\mathcal{I}_{4}\end{aligned} \tag{58}$$

where a sign has been added for a ghost loop. It is seen that the total contribution to the effective action is nonzero $\left\langle S_{GH2}\right\rangle_{4}-\frac{1}{2}\left\langle S_{GH1}^{2}\right\rangle_{4}\neq 0$, which leads to a divergent mass term and violates $U(1)$ gauge invariance without imposing proper regularization schemes to treat such a quartic divergence. Obviously, in the cut-off regularization, $\mathcal{I}_{4}^{R}$ is proportional to Λ^{4}, which then destroys the gauge invariance. While the LORE method is found to be a proper regularization scheme as it leads $\mathcal{I}_{4}^{R}=0$, so that the regularized quartic divergence disappears and the gauge invariance is maintained. Though the dimensional regularization results in $\mathcal{I}_{4}^{R}=0$, while it also gives $\mathcal{I}_{2}^{R}=0$.

IV. CONCLUSIONS

In summary, we have investigated the one-loop quadratically divergent gravitational corrections to gauge Green's functions both in diagrammatic calculation and in the gauge condition independent Vilkovisky-DeWitt background field method. As a consequence, we have obtained in both cases the quadratically divergent gravitational contributions to the β function of gauge coupling constant. We limit our discussion in one-loop approximation. This approximation can break down as approaching the Planck scale, where new framework of quantum gravity is needed.

In the diagrammatic approach, we have explicitly performed the calculations for the two-, three- and four-point gauge Green's functions at one-loop level with graviton contributions.

We have demonstrated for the first time that the Slavnov-Taylor identities are satisfied for these gravitational corrections, which is found to be irrespective of the regularization schemes. However, our analysis has shown that the gravitational contribution to the β function is dependent on the regularization schemes in the harmonic gauge condition. Both the cut-off and dimensional regularization schemes lead to a zero result, here the former is due to the accidental cancelation with the inconsistency relation $\mathcal{I}^R_{2\mu\nu} = \frac{1}{4} g_{\mu\nu} \mathcal{I}^R_2$ which spoils gauge invariance, and the latter is due to the well-known suppression effect of dimensional regularization to the quadratic divergence with $\mathcal{I}^R_2 = 0$. In contrast, the LORE method gives a non-zero result that render all gauge theory asymptotically free at very high energy scale, this is because the LORE method preserves the gauge symmetry and maintains the quadratic divergent behavior with the consistency condition $\mathcal{I}^R_{2\mu\nu} = \frac{1}{4} g_{\mu\nu} \mathcal{I}^R_2$.

In the second part of our calculations, we have worked within the framework of Vilkovisky-DeWitt's effective action which is gauge-condition independent. We have shown in a regularization scheme independent way that there is in general quadratically divergent gravitational contributions to the gauge coupling. It is interesting to notice that when reducing to the framework of traditional background field approach with harmonic gauge condition, we arrive at the same results obtained in the diagrammatic approach, which shows the equivalence between the diagrammatic approach and traditional background method. We have found that in any case the symmetry-maintaining and divergent-behavior-preserving LORE method leads to an asymptotic free power-law running of gauge coupling at one-loop near the Planck scale due to quantum gravitational contributions. In particular, we have paid attention to the treatment on the quartic divergent effect which in general violates gauge invariance, again the LORE method is found to be a proper regularization scheme to handle the quartic divergence for ensuring gauge invariance.

Note added: At the final stage of this work, there is a preprint [42], which also discussed the Einstein-Maxwell system, Ward identities and Vilkovisky-DeWitt's formalism. Part of our calculation on quadratic divergence is confirmed by [42] using proper-time representation.

Acknowledgement

This work was supported in part by the National Science Foundation of China (NSFC) under Grant #No. 10821504, 10975170 and the key project of the Chinese Academy of Sciences.

Appendix

In the following appendix sections, we are going to present a concise overview for the traditional and Vilkovisky-DeWitt's modified effective action for completeness and convenience. The introduction of traditional effective action can be found in the standard textbook[29]. The original idea of Vilkovisky-DeWitt's effective action was presented in[9, 10]. For a pedagogical review on this topic, the readers are referred to the article[34] and references therein.

Appendix A: Traditional effective action

We begin with a brief description of the traditional approach to define an effective action for non-gauge theories. Let $S[\varphi]$ be the classical action for the theory, then the generating functional $Z[J]$ in the presence of external currents for the n-point Green's functions is defined by

$$Z[J] = \mathcal{N} \int \mathcal{D}\varphi \exp i \left\{ S[\varphi] + \varphi^i J_i \right\} \tag{A1}$$

where the functional integration measure is

$$\mathcal{D}\varphi = \left(\prod_i d\varphi^i \right) \tag{A2}$$

$\mathcal{N}$ is an irrelevant constant for normalization and will be neglected below. $Z[J]$ has a physical diagrammatic picture that it is the the sum of all vacuum-to-vacuum amplitudes, including both disconnected and connected diagrams. For the connected Green's functions, it is useful to define another generating functional, $W[J]$, with

$$W[J] = -i \ln Z[J] \tag{A3}$$

For One-Particle-Irreducible(OPI) diagrams, one can go one step further. Define the background fields $\bar{\varphi}^i$ as

$$\bar{\varphi}^i(x) \ \equiv \ \frac{\langle \mathrm{out} \, |\varphi^i(x)| \, \mathrm{in} \rangle_J}{\langle \mathrm{out}|\mathrm{in} \rangle_J} = \frac{-i}{Z[J]} \frac{\delta Z[J]}{\delta J^i(x)} = \frac{\delta W[J]}{\delta J^i(x)} \tag{A4}$$

Now, the quantum effective action $\Gamma[\bar{\varphi}]$ is defined by the Legendre transformation of $W[J]$,

$$\Gamma[\bar{\varphi}] \equiv W[J] - \bar{\varphi}^i J_i \tag{A5}$$

It can be shown that $\bar{\varphi}^i$ satisfies the equation

$$\frac{\delta\Gamma[\bar{\varphi}]}{\delta\bar{\varphi}^i(x)} = -J_i(x) \tag{A6}$$

and $\Gamma[\bar{\varphi}]$ is the generating functional for OPI Green functions. In the functional integral representation, we have

$$\begin{aligned}
\exp i\Gamma[\bar{\varphi}] &= \int \mathcal{D}\varphi \exp i\left\{S[\varphi] + \left(\varphi^i - \bar{\varphi}^i\right) J_i\right\} \\
&= \int \mathcal{D}\varphi \exp i\left\{S[\varphi] - \left(\varphi^i - \bar{\varphi}^i\right) \frac{\delta\Gamma[\bar{\varphi}]}{\delta\bar{\varphi}^i}\right\}
\end{aligned} \tag{A7}$$

Both sides of the above equation have $\Gamma[\bar{\varphi}]$, so it will involve an iterative procedure to solve the equation in perturbative expansion. For example, to get $\Gamma[\bar{\varphi}]$ at one loop level on the Left-Hand-Side(LHS), we can replace the $\Gamma[\bar{\varphi}]$ on the Right-Hand-Side(RHS) with its tree level value $S[\bar{\varphi}]$.

In the background field approach, one expands the fields φ^i as the sum of background fields $\bar{\varphi}^i$ and quantum fields η^i,

$$\varphi^i = \bar{\varphi}^i + \eta^i. \tag{A8}$$

To get the one-loop effective action, only the quadratic terms of η^i in the exponent need to be kept,

$$\begin{aligned}
\exp i\Gamma[\bar{\varphi}] &= \int \mathcal{D}\eta \exp i\left\{S[\bar{\varphi}^i + \eta^i] - \eta^i \frac{\delta S[\bar{\varphi}]}{\delta\bar{\varphi}^i}\right\} \\
&= \exp iS[\bar{\varphi}] \int \mathcal{D}\eta \exp i\left\{\frac{1}{2}\eta^i \frac{\delta S[\bar{\varphi}]}{\delta\bar{\varphi}^i\delta\bar{\varphi}^j}\eta^j\right\}
\end{aligned}$$

then, the effective action $\Gamma[\bar{\varphi}]$ is given by

$$\Gamma[\bar{\varphi}] \;=\; S[\bar{\varphi}] + \frac{i}{2}\ln\det S_{,ij} \tag{A9}$$

Appendix B: The Vilkovisky-DeWitt effective action

The above formalism for defining quantum effective action has the problem that it depends on the parametrization of $\varphi' = \varphi'(\varphi)$ [9]. Suppose $S[\varphi]$ is a scalar under the transformation $\varphi' = \varphi'(\varphi)$, so should be expected for $\Gamma[\varphi]$. However, in a different parametrization $\varphi' = \varphi'(\varphi)$, the effective action will be changed [9]

$$\Gamma'[\bar{\varphi'}] = S'[\bar{\varphi}'] + \frac{i}{2}\ln\det\left[S_{,ij} + S_{,k}\frac{\partial^2\varphi^k}{\partial\varphi'^l\partial\varphi'^m}\frac{\partial\varphi'^l}{\partial\varphi^i}\frac{\partial\varphi'^m}{\partial\varphi^j}\right] \tag{B1}$$

Since $S'[\bar{\varphi}'] = S[\bar{\varphi}]$, eq.(B1) will be different from eq.(A9) at one loop level already. To solve the field parametrization dependence, Vilkovisky [9] suggested that we might regard the field space φ^i as a manifold M which is associated with the metric $g_{ij}[\varphi]$, connection Γ^i_{jk}, treat the field φ^i as the coordinates on this manifold, and define the effective action in terms of parametrization invariant quantities. Later, DeWitt discussed this issue further in [10]. In the following, we shall follow the discussion in ref. [34] where a complete and clear discussion was given.

Define the two-point function or world function [35],

$$\sigma[\varphi_\star;\varphi] = \frac{1}{2}(\text{length of geodesic from } \varphi_\star \text{to } \varphi)^2 \tag{B2}$$

and

$$\sigma^i[\varphi_\star;\varphi] = g^{ij}[\varphi_\star]\frac{\delta\sigma[\varphi_\star;\varphi]}{\delta\varphi^j_\star} \tag{B3}$$

$\sigma^i[\varphi_\star;\varphi]$ is a vector and tangent to the geodesic line on the field space that connects $\varphi_\star$ and φ. $\sigma^i[\varphi_\star;\varphi]$ can be expanded in powers of $\eta^i = \varphi^i - \bar{\varphi}^i$,

$$\sigma^i[\varphi_\star;\varphi] = -\eta^i + \sum_{n=2}^{\infty}\frac{1}{n!}\sigma^i{}_{j_1\cdots j_n}\eta^{j_1}\cdots\eta^{j_n},\ \sigma^i{}_{jk} = \Gamma^i_{jk} \tag{B4}$$

where $\varphi_\star$ is an arbitrary point at the moment. Then the generating functional eq.(A1) is modified to be

$$Z[J] \;=\; \int d\mu[\varphi_\star;\varphi]\exp i\left\{S[\varphi] - J_i\sigma^i[\varphi_\star;\varphi]\right\} = \exp iW[J;\varphi_\star] \tag{B5}$$

where the measure

$$d\mu[\varphi_\star;\varphi] \;=\; \left(\prod_i d\sigma^i[\varphi_\star;\varphi]\right)\sqrt{|g[\varphi_\star]|} = \left(\prod_i d\varphi^i\right)\sqrt{|g[\varphi]|}\,|J[\varphi_\star;\varphi]|$$

$J[\varphi_\star;\varphi]$ is the Jacobean factor, which is irrelevant at one loop order. $S[\varphi]$ is treated as functional of $\varphi_\star$ and $\sigma^i[\varphi_\star;\varphi]$, $\hat{S}\left[\varphi_\star;\sigma^i[\varphi_\star;\varphi]\right]$ is defined by covariant Taylor expansion,

$$S[\varphi] = \hat{S}\left[\varphi_\star;\sigma^i[\varphi_\star;\varphi]\right] = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}S_{;i_1\cdots i_n}[\varphi_\star]\sigma^{i_1}[\varphi_\star;\varphi]\cdots\sigma^{i_n}[\varphi_\star;\varphi] \tag{B6}$$

then $Z[J]$ of eq.(B5) can be regarded as the generating functional for Green function of $\sigma^i[\varphi_\star;\varphi]$, but we are more interested in Green function of φ^i. Define $\bar{\varphi}$ and v^i through

$$v^i \equiv \sigma^i[\varphi_\star;\bar{\varphi}] \equiv \left\langle\sigma^i[\varphi_\star;\varphi]\right\rangle = \frac{\delta W[J;\varphi_\star]}{\delta J_i}. \tag{B7}$$

Similarly, the corresponding effective action then is

$$\hat{\Gamma}\left[\varphi_\star;\sigma^i[\varphi_\star;\bar{\varphi}]\right]=W[J;\varphi_\star]+J_i\sigma^i[\varphi_\star;\bar{\varphi}], \tag{B8}$$

$$\exp i\hat{\Gamma}\left[\varphi_\star;\sigma^i[\varphi_\star;\bar{\varphi}]\right]=\int d\mu[\varphi_\star;\varphi]\exp i\left\{S[\varphi]-\frac{\delta\hat{\Gamma}}{\delta v^i}\left(\sigma^i[\varphi_\star;\varphi]-v^i\right)\right\}. \tag{B9}$$

By expanding the functional $\hat{S}\left[\varphi_\star;\sigma^i[\varphi_\star;\varphi]\right]$ at $\sigma^i=v^i$

$$\hat{S}\left[\varphi_\star;\sigma^i[\varphi_\star;\varphi]\right]=\hat{S}\left[\varphi_\star;v^i\right]+\sum_{n=0}^{\infty}\frac{1}{n!}\frac{\delta^n\hat{S}\left[\varphi_\star;v^i\right]}{\delta v^{i_1}\cdots\delta v^{i_n}}(\sigma^{i_1}-v^{i_1})\cdots(\sigma^{i_n}-v^{i_n}), \tag{B10}$$

where the expansion coefficients are connected to those of covariant Taylor expansion,

$$\frac{\delta^n\hat{S}\left[\varphi_\star;v^i\right]}{\delta v^{i_1}\cdots\delta v^{i_n}}=(-1)^nS_{;i_1\cdots i_n}[\varphi_\star]+\sum_{m=n+1}^{\infty}\frac{(-1)^m}{(m-n)!}S_{;i_1\cdots i_m}[\varphi_\star]v^{i_1}\cdots v^{i_m} \tag{B11}$$

Then at one loop level, the effective action is given by

$$\hat{\Gamma}\left[\varphi_\star;\sigma^i[\varphi_\star;\bar{\varphi}]\right]\;=\;\hat{S}\left[\varphi_\star;\sigma^i[\varphi_\star;\bar{\varphi}]\right]+\frac{i}{2}\ln\det\left[g^{ik}[\varphi_\star]\frac{\delta^2\hat{S}\left[\varphi_\star;v^i\right]}{\delta v^k\delta v^j}\right], \tag{B12}$$

When $\varphi^\star$ is arbitrary, we can take the limit $\varphi_\star\to\bar{\varphi}$ and $v^i\to 0$, then we have

$$\left.\frac{\delta^n\hat{S}\left[\varphi_\star;v^i\right]}{\delta v^{i_1}\cdots\delta v^{i_n}}\right|_{v=0}=(-1)^nS_{;i_1\cdots i_n}, \tag{B13}$$

and the effective action

$$\begin{aligned}\Gamma[\bar{\varphi}]=\hat{\Gamma}\left[\bar{\varphi};\sigma^i[\bar{\varphi};\bar{\varphi}]\right]\;&=\;\hat{S}\left[\bar{\varphi};\sigma^i[\bar{\varphi};\bar{\varphi}]\right]+\frac{i}{2}\ln\det\left[g^{ik}[\varphi_\star]\frac{\delta^2\hat{S}\left[\varphi_\star;v^i\right]}{\delta v^k\delta v^j}\right]_{v=0}\\&=\;S[\bar{\varphi}]+\frac{i}{2}\ln\det\left[\nabla^i\nabla_jS[\bar{\varphi}]\right]\end{aligned} \tag{B14}$$

The above formalism can be generalized to multi-loops [34].

For gauge theories, some modifications are needed. Let $S[\varphi]$ represent the classical action functional for a gauge theory, it is gauge invariant under the transformation

$$\delta\varphi^i=K^i_\alpha[\varphi]\delta\epsilon^\alpha. \tag{B15}$$

with $K^i_\alpha[\varphi]$ regarded as the generators of gauge transformations and $\delta\epsilon^\alpha$ infinitesimal parameters. $S[\varphi]$ is gauge invariant in the sense of

$$S_{,i}\delta\varphi^i=S_{,i}K^i_\alpha[\varphi]\delta\epsilon^\alpha=0,\forall\quad\delta\epsilon^\alpha\Longrightarrow S_{,i}K^i_\alpha[\varphi]=0 \tag{B16}$$

To quantize gauge theory, a gauge fixing condition has to be imposed, for instance, $\chi^\alpha[\varphi] = f^\alpha$, where f^α is independent of φ^i. Since we want to fix the gauge field, the gauge condition then should not be gauge invariant. Then require $\chi^\alpha[\varphi+\delta\varphi] = \chi^\alpha[\varphi]$ hold only if $\delta\epsilon^\alpha = 0$, one has

$$\chi^\alpha{}_{,i}[\varphi]K^i_\beta[\varphi]\delta\epsilon^\beta \equiv Q^\alpha{}_\beta[\varphi]\delta\epsilon^\beta = 0 \tag{B17}$$

Then $\det Q^\alpha{}_\beta$ is usually called as the Faddeev-Popov factor [36]. In Path-Integral quantization, for a field space M that has gauge symmetry G, we only have to integrate the gauge nonequivalent field configuration on the reduced field space $\mathcal{M} = M/G$ for the generating functional, and use $\tilde{g}_{ij}$ and $\tilde{\Gamma}^i_{jk}$ on $\mathcal{M}$ to define the gauge and parametrization invariant effective action. However, it is usually more convenient to work in the whole field space M by inserting Faddeev-Popov factor $\det Q^\alpha{}_\beta$ and gauge condition $\delta\left[\chi^\alpha - f^\alpha\right]$ into the integral measure and expressing $\tilde{g}_{ij}$ and $\tilde{\Gamma}^i_{jk}$ on $\mathcal{M}$ in terms of g_{ij} and Γ^i_{jk} on M, with [9, 10]

$$\begin{aligned}
\tilde{g}_{ij} &= g_{ij} - K_{i\alpha}\gamma^{\alpha\beta}K_{j\beta},\ \gamma_{\alpha\beta} = K^i_\alpha g_{ij}K^j_\beta && \text{(B18)}\\
\tilde{\Gamma}^k_{ij} &= \Gamma^k_{ij} + T^k_{ij} + K^k_\alpha A^\alpha_{ij} && \text{(B19)}\\
T^k_{ij} &= \frac{1}{2}\gamma^{\alpha\lambda}\gamma^{\beta\tau}K_{\alpha i}K_{\beta j}(K^n_\lambda K^k_{\tau;n} + K^n_\tau K^k_{\lambda;n})\\
&\quad -\gamma^{\alpha\beta}(K_{\alpha i}K^k_{\beta;j} + K_{\alpha j}K^k_{\beta;i})
\end{aligned}$$

where $A^\alpha_{ij} = A^\alpha_{ji}$ is arbitrary. $\tilde{\Gamma}^k_{ij}$ is different from Γ^k_{ij} by two additional terms, T^k_{ij} and $K^k_\alpha A^\alpha_{ij}$. It will be shown later that $K^k_\alpha A^\alpha_{ij}$ term will not contribute to one loop effective action because of the gauge invarance of S and then to any order Γ by induction. Choose a gauge fixing condition, $\chi^\alpha = f^\alpha$, then the effective action is

$$\exp i\hat{\Gamma}\left[\varphi_\star; v^i\right] = \int d\mu[\varphi_\star;\varphi]\delta\left[\chi^\alpha - f^\alpha\right]\det Q^\alpha{}_\beta \exp i\left[S[\varphi] - \frac{\delta\hat{\Gamma}}{\delta v^i}\left(\sigma^i[\varphi_\star;\varphi] - v^i\right)\right] \tag{B20}$$

Physical results should be independent of this choice of f^α, so we can insert the integration with $\int \mathcal{D}f^\alpha \exp\left[\frac{i}{2\Omega}f^\alpha f_\alpha\right]$ and do a Gaussian average over f^α. Integrate f^α first and use $\delta\left[\chi^\alpha - f^\alpha\right]$, $\exp\left[\frac{i}{2\Omega}f^\alpha f_\alpha\right]$ is turned into $\exp\left[\frac{i}{2\Omega}\chi^\alpha\chi_\alpha\right]$, then at one-loop approximation the effective action is given by

$$\Gamma[\bar{\varphi}] = S[\bar{\varphi}] - i\ln\det Q^\alpha{}_\beta + \frac{i}{2}\ln\det\left(\nabla^i\nabla_j S[\bar{\varphi}] + \frac{1}{2\Omega}\chi_\alpha{}^{,i}\chi^\alpha{}_{,j}\right) \tag{B21}$$

where $\nabla_i\nabla_j S[\bar{\varphi}] = S_{,ij}[\bar{\varphi}] - \tilde{\Gamma}^k_{ij}S_{,k}[\bar{\varphi}]$. The corresponding effective action with Euclidean metric is

$$\Gamma[\bar{\varphi}] = S[\bar{\varphi}] - \ln\det Q^\alpha{}_\beta + \frac{1}{2}\ln\det\left(\nabla^i\nabla_j S[\bar{\varphi}] + \frac{1}{2\Omega}\chi_\alpha{}^{,i}\chi^\alpha{}_{,j}\right) \tag{B22}$$

To calculate the above effective action, one can either use the standard procedure, $\ln\det D \to \mathrm{Tr}\ \ln D$ and expand $\ln D$ in series, or equivalently rewrite the determinant back to the functional integration [25],

$$\Gamma_G = \frac{1}{2}\ln\det\left(\nabla^i\nabla_j S[\bar{\varphi}] + \frac{1}{2\Omega}\chi_\alpha{}^{,i}\chi^\alpha{}_{,j}\right) = -\ln\int \mathcal{D}\eta e^{-S_q} \tag{B23}$$

$$S_q = \frac{1}{2}\eta^i\eta^j\left[S_{,ij} - \tilde{\Gamma}^k_{ij}S_{,k} + \frac{1}{2\Omega}\chi_\alpha{}^{,i}\chi^\alpha{}_{,j}\right] \tag{B24}$$

$$\Gamma_{GH} = -\ln\det Q_{\alpha\beta} = -\ln\int\left[\mathcal{D}\bar{c}\mathcal{D}c\right]e^{-S_{GH}}, \tag{B25}$$

with $S_q = S_0 + S_1 + S_2$ in eq.(B23) and $S_{GH} = \bar{\eta}_\alpha Q^\alpha{}_\beta\eta^\beta = S_{GH0} + S_{GH1} + S_{GH2}$ in eq.(B25). The subscripts on S denotes the order in the background field $\bar{\varphi}$. Γ_{GH} is the ghost contribution with $\bar{c}_\alpha$ and c^β are anti-commuting ghost fields. At one loop order, we have

$$\begin{aligned}\Gamma_G &= -\ln\int \mathcal{D}\eta e^{-S_0-S_1-S_2} = -\ln\int\mathcal{D}\eta\left[1 - S_2 + \frac{1}{2}S_1^2\right]e^{-S_0} \\ &= -\ln\int\mathcal{D}\eta e^{-S_0} + \frac{\int\mathcal{D}\eta\left[S_2 - \frac{1}{2}S_1^2\right]e^{-S_0}}{\int\mathcal{D}\eta e^{-S_0}} \\ &\approx \langle S_2\rangle - \frac{1}{2}\left\langle S_1^2\right\rangle \end{aligned}\tag{B26}$$

where $\approx$ means that we have ignored the irrelevant infinite constant. Similarly, for the ghost's part, we have

$$\begin{aligned}\Gamma_{GH} &= -\ln\int\left[\mathcal{D}\bar{c}\mathcal{D}c\right]e^{-S_{GH0}-S_{GH1}-S_{GH2}} \\ &\approx \langle S_{GH2}\rangle - \frac{1}{2}\left\langle S_{GH1}^2\right\rangle\end{aligned}\tag{B27}$$

Note that the connection terms $T^k_{ij}S_{,k}$ and $A_{ij}K^k_\alpha S_{,k}$ in eq.(B23) can be written as [40]

$$S_T = -\frac{1}{2}\eta^i T^k_{ij}S_{,k}\eta^j = (\eta^i K^\beta_i)K^k_{\beta;j}S_{,k}(\eta^j - \frac{1}{2}\eta^l K^\alpha_l K^j_\alpha) \tag{B28}$$

$$S_K = -\frac{1}{2}\eta^i A_{ij}K^k_\alpha S_{,k}\eta^j = 0,\ \text{since}\ K^k_\alpha S_{,k} = 0 \tag{B29}$$

For the sake of S_T, we will work in Landau-DeWitt gauge condition [37] which has the following feature and can simplify the calculation significantly

$$\chi_\alpha = K_{\alpha i}[\bar{\varphi}]\eta^i = 0 \Longrightarrow S_T = 0 \tag{B30}$$

This means that the difference between $\tilde{\Gamma}^k_{ij}$ and Γ^k_{ij} does not contribute to the effective action at one loop. For multi-loop result, this is only true for special case that the metric g_{ij} doesn't

depend on the field φ^i [34, 37]. In this gauge, we can use the representation of δ-function,

$$\delta[\chi^\alpha] = \lim_{\Omega\to 0}\left[\det\left(\frac{\delta_{\alpha\beta}}{4\pi\Omega}\right)\right]^{\frac{1}{2}}\exp\left[-\frac{1}{2\Omega}\chi^\alpha\chi_\alpha\right] \tag{B31}$$

Then at one-loop order with Landau-DeWitt gauge, the effective action is given by

$$\begin{aligned}\Gamma[\bar\varphi] &= S[\bar\varphi] - \ln\det Q_{\alpha\beta}[\bar\varphi] \\ &\quad +\frac{1}{2}\lim_{\Omega\to 0}\ln\det\left(\nabla^i\nabla_j S[\bar\varphi] + \frac{1}{2\Omega}K^i_\alpha[\bar\varphi]K^\alpha_j[\bar\varphi]\right)\end{aligned} \tag{B32}$$

with $\nabla_i\nabla_j S[\bar\varphi] = S_{,ij}[\bar\varphi] - \Gamma^k_{ij}S_{,k}[\bar\varphi]$, here the Christoffel connection Γ^k_{ij} is determined by $g_{ij}[\varphi]$. Note that if any other gauge condition is chosen, Eq. (B32) will not be true and the complicated form will replace it with the full $\tilde\Gamma^k_{ij}$, Eq.(B19). It is noticed that the connection term $\Gamma^k_{ij}S_{,k}[\bar\varphi]$ distinguishes the Vilkovisky-DeWitt's method from the traditional background-field method. Also, Ω in Landau-DeWitt gauge has to be enforced to 0 at the end of calculation since it has a different origin from the Ω in Eq.(B22).

In this appendix, we shall give the useful formula in our calculation. We also show the details of our computation of the Christoffel connection in the field space, Γ^i_{ij} and the functional derivatives, $S_{,i}$ and $S_{,ij}$ in a general background space-time. The classical action functional of Einstein-Maxwell theory with Euclidean metric is

$$S = S_M + S_G = \int d^4x|g(x)|^{\frac{1}{2}}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{2}{\kappa^2}(R-2\Lambda)\right], \tag{B33}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\kappa^2 = 32\pi G$, G is the Newton's gravitational constant, Λ is the cosmological constant, and

$$S_M = \frac{1}{4}\int d^4x|g(x)|^{\frac{1}{2}}F_{\mu\nu}F^{\mu\nu}, \tag{B34}$$

$$S_G = -\frac{2}{\kappa^2}\int d^4x|g(x)|^{\frac{1}{2}}(R-2\Lambda), \tag{B35}$$

and Riemann tensor

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho_{\nu\sigma} - \partial_\nu\Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma}, \tag{B36}$$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2}g^{\rho\lambda}\left[\frac{\partial g_{\mu\lambda}}{\partial x^\nu} + \frac{\partial g_{\nu\lambda}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right]. \tag{B37}$$

Although we use the same symbol $\Gamma^\rho_{\mu\nu}$, it should not be confused with the connection $\Gamma^i_{jk}[\varphi^i]$ on the field space. The action eq.(B33) is invariant under general coordinate and $U(1)$ gauge

transformations,

$$
\begin{aligned}
\delta g_{\mu\nu} &= -\delta\epsilon^\rho g_{\mu\nu,\rho} - \delta\epsilon^\rho{}_{,\mu} g_{\rho\nu} - \delta\epsilon^\rho{}_{,\nu} g_{\rho\mu}, &\text{(B38)}\\
\delta A_\mu &= -\delta\epsilon^\nu A_{\mu,\nu} - \delta\epsilon^\nu{}_{,\mu} A_\nu + \delta\epsilon_{,\mu}. &\text{(B39)}
\end{aligned}
$$

Both the general coordinate and $U(1)$ gauge transformations affect the gauge field, shown above. Write the above transformations in the form of $\delta\varphi^i = K^i_\alpha \delta\epsilon^\alpha$ where $\varphi^i = (g_{\mu\nu},\ A_\mu)$ and $\epsilon^i = (\epsilon^\mu, \epsilon)$, explicitly, we have

$$
\begin{aligned}
\delta g_{\mu\nu}(x) &= \int d^4x' \left[K^{g_{\mu\nu}(x)}{}_\rho(x,x')\delta\epsilon^\rho(x') + K^{g_{\mu\nu}(x)}(x,x')\delta\epsilon(x')\right], &\text{(B40)}\\
\delta A_\mu(x) &= \int d^4x' \left[K^{A_\mu(x)}{}_\rho(x,x')\delta\epsilon^\rho(x') + K^{A_\mu(x)}(x,x')\delta\epsilon(x')\right]. &\text{(B41)}
\end{aligned}
$$

The generators K^i_α for symmetric transformations defined above are given by

$$
\begin{aligned}
K^{g_{\mu\nu}(x)}{}_\rho(x,x') &= \left[-g_{\mu\nu,\rho}(x) - 2g_{\rho(\mu}(x)\partial_{\nu)}\right]\delta(x,x') &\text{(B42)}\\
K^{g_{\mu\nu}(x)}(x,x') &= 0 &\text{(B43)}\\
K^{A_\mu(x)}{}_\rho(x,x') &= \left[-A_{\mu,\rho}(x) - A_\rho(x)\partial_\mu\right]\delta(x,x') &\text{(B44)}\\
K^{A_\mu(x)}(x,x') &= \partial_\mu\delta(x,x') &\text{(B45)}
\end{aligned}
$$

The parentheses mean the symmetrization over enclosed indices. $\delta(x,x')$ has the following features

$$
\begin{aligned}
\int d^4x' F(x')\delta(x,x') &= F(x)\\
\int d^4x' F(x')\partial_\mu\delta(x,x') &= \partial_\mu F(x)\\
\int d^4x' F(x')\partial_\mu\partial_\nu\delta(x,x') &= \partial_\mu\partial_\nu F(x) &\text{(B46)}
\end{aligned}
$$

The explicit form of $\delta(x,x')$ is not important here, all we need in the calculation are the features above. It can be shown that $\delta(x,x')$ of the following form can satisfy the above features

$$
\delta(x,x') = |g(x')|^{\frac{1}{2}}\delta(x-x')|g(x)|^{-\frac{1}{2}} \text{ or simply } \delta(x-x')
$$

where $\delta(x-x')$ is the usual Dirac δ-function in flat space-time. We will not rely on this explicit form of $\delta(x,x')$ in the calculations.

Now we should choose a proper metric on the field space. At first sight, the metric seems arbitrary. It is suggested in [9, 38] that there are several guidelines or rules for the choice of the metric being unique, the effects of metric have been discussed in [39] when relaxing one of the rules. The metric on the field space φ^i, G_{ij}, can be defined by the line element, $ds^2 = G_{ij} d\varphi^i d\varphi^j$,

$$ds^2 = \int d^n x d^n x' \left\{ G_{g_{\mu\nu}(x) g_{\rho\sigma}(x')} dg_{\mu\nu}(x) dg_{\rho\sigma}(x') + G_{A_\mu(x) A_\nu(x')} dA_\mu(x) dA_\nu(x') \right\} \tag{B47}$$

where the metric has the following form [25]

$$G_{g_{\mu\nu}(x) g_{\rho\sigma}(x')} = \frac{1}{\kappa^2} |g(x)|^{\frac{1}{2}} \left(g^{\mu(\rho} g^{\sigma)\nu} - \frac{1}{2} g^{\mu\nu} g^{\rho\sigma} \right) \delta(x, x') \tag{B48}$$

$$G_{A_\mu(x) A_\nu(x')} = |g(x)|^{\frac{1}{2}} g^{\mu\nu}(x) \delta(x, x') \tag{B49}$$

The inverse metric is

$$G^{g_{\mu\nu}(x) g_{\rho\sigma}(x')} = \kappa^2 |g(x)|^{-\frac{1}{2}} \left(g_{\mu(\rho} g_{\sigma)\nu} - \frac{1}{2} g_{\mu\nu} g_{\rho\sigma} \right) \delta(x, x'). \tag{B50}$$

$$G^{A_\mu(x) A_\nu(x')} = |g(x)|^{-\frac{1}{2}} g_{\mu\nu}(x) \delta(x, x'). \tag{B51}$$

The orthogonal relation $G^{ij} G_{jk} = \delta^i_k$ reads explicitly as

$$\int d^4 x' G^{g_{\mu\nu}(x) g_{\rho\sigma}(x')} G_{g_{\rho\sigma}(x') g_{\lambda\tau}(x'')} = \delta^\lambda_{(\mu} \delta^\tau_{\nu)} \delta(x, x'') \tag{B52}$$

$$\int d^4 x' G^{A_\mu(x) A_\nu(x')} G_{A_\nu(x') A_\rho(x'')} = \delta^\rho_\mu \delta(x, x'') \tag{B53}$$

Using the metric and inverse metric on the field space, we can determine the corresponding Christoffel connection through

$$\Gamma^k_{ij} = \frac{1}{2} G^{kl} \left[\frac{\delta G_{il}}{\delta \varphi^j} + \frac{\delta G_{jl}}{\delta \varphi^i} - \frac{\delta G_{ij}}{\delta \varphi^l} \right] \tag{B54}$$

Below, we show the details of the tedious calculation for the Γ^k_{ij}.

Appendix C: Christoffel Connection on the field space

In this section, we present the details to calculate the Christoffel Connection on the field space [25]. Some useful formula for derivation are listed below

$$\frac{\delta g_{\mu\nu}(x)}{\delta g_{\rho\sigma}(x')} = \delta^\rho_{(\mu}\delta^\sigma_{\nu)}\delta(x,x'),\ \delta^\rho_{(\mu}\delta^\sigma_{\nu)} = \frac{1}{2}\left[\delta^\rho_\mu\delta^\sigma_\nu + \delta^\rho_\nu\delta^\sigma_\mu\right] \tag{C1}$$

$$\frac{\delta A_\mu(x)}{\delta A_\nu(x')} = \delta^\nu_\mu\delta(x,x'),\ \frac{\delta g_{\mu\nu}(x)}{\delta A_\rho(x')} = 0 = \frac{\delta A_\rho(x)}{\delta g_{\mu\nu}(x')},\ \frac{\delta[\delta(x,x')]}{\delta\varphi^i} = 0 \tag{C2}$$

$$\delta g^{\mu\nu}(x) = -g^{\mu\rho}(x)g^{\nu\sigma}(x)\delta g_{\rho\sigma}(x),\ \frac{\delta g^{\mu\nu}(x)}{\delta g_{\rho\sigma}(x')} = -g^{\mu(\rho}g^{\sigma)\nu}\delta(x,x') \tag{C3}$$

$$\delta|g(x)|^{\frac{1}{2}} = \frac{1}{2}|g(x)|^{\frac{1}{2}}g^{\rho\sigma}(x)\delta g_{\rho\sigma}(x),\ \frac{\delta|g(x)|^{\frac{1}{2}}}{\delta g_{\rho\sigma}(x')} = \frac{1}{2}|g(x)|^{\frac{1}{2}}g^{\rho\sigma}(x)\delta(x,x') \tag{C4}$$

We can calculate the first non-zero component of Christoffel connection,

$$\begin{aligned}\Gamma^{g_{\mu\nu}(x)}_{A_\lambda(x')A_\tau(x'')} &= \int d^4\bar{x}\frac{1}{2}G^{g_{\mu\nu}(x)g_{\rho\sigma}(\bar{x})}\left[-\frac{\delta G_{A_\lambda(x')A_\tau(x'')}}{\delta g_{\rho\sigma}(\bar{x})}\right]\\ &= \frac{1}{2}\kappa^2\delta^\lambda_{(\mu}\delta^\tau_{\nu)}\delta(x,x')\delta(x',x'')\end{aligned} \tag{C5}$$

Note that there is a missing κ^2 in the corresponding equation in [25], but the final expanded action there includes the κ^2 back. In deriving the above equation, we have used $G^{g_{\mu\nu}(x)A_\rho(\bar{x})} = 0$, $G_{g_{\mu\nu}(x)g_{\rho\sigma}(\bar{x}),A_\tau(x')} = 0$ and

$$\frac{\delta\left(|g(x')|^{\frac{1}{2}}g^{\lambda\tau}(x')\right)}{\delta g_{\rho\sigma}(\bar{x})} = -|g(x')|^{\frac{1}{2}}\left[g^{\lambda(\rho}g^{\sigma)\tau} - \frac{1}{2}g^{\rho\sigma}g^{\lambda\tau}\right]\delta(x',\bar{x}) \tag{C6}$$

The next non-vanishing component is

$$\begin{aligned}\Gamma^{A_\mu(x)}_{A_\nu(x')g_{\alpha\beta}(x'')} &= \int d^4\bar{x}\frac{1}{2}G^{A_\mu(x)A_\lambda(\bar{x})}\left[\frac{\delta G_{A_\nu(x')A_\lambda(\bar{x})}}{\delta g_{\alpha\beta}(x'')}\right]\\ &= \frac{1}{4}\left[g^{\alpha\beta}\delta^\nu_\mu - 2g^{\nu(\alpha}\delta^{\beta)}_\mu\right]\delta(x,x')\delta(x',x'') = \Gamma^{A_\mu(x)}_{g_{\alpha\beta}(x'')A_\nu(x')}\end{aligned} \tag{C7}$$

and the most complicated component is

$$\begin{aligned}&\Gamma^{g_{\lambda\tau}(x)}_{g_{\mu\nu}(x')g_{\rho\sigma}(x'')}\\ &= \int d^4\bar{x}\frac{1}{2}G^{g_{\lambda\tau}(x)g_{\alpha\beta}(\bar{x})}\left[\frac{\delta G_{g_{\mu\nu}(x')g_{\alpha\beta}(\bar{x})}}{\delta g_{\rho\sigma}(x'')} + \frac{\delta G_{g_{\alpha\beta}(\bar{x})g_{\rho\sigma}(x'')}}{\delta g_{\mu\nu}(x')} - \frac{\delta G_{g_{\mu\nu}(x')g_{\rho\sigma}(x'')}}{\delta g_{\alpha\beta}(\bar{x})}\right]\end{aligned} \tag{C8}$$

This quantity is well-known in the literature, for instance [37], here we show the details as a check of our calculation. To calculate the above expression, using eq.(B48) and eq.(C4),

we can work out the first term in the bracket

$$\begin{aligned}
\frac{\delta G_{g_{\mu\nu}(x')g_{\rho\sigma}(x'')}}{\delta g_{\alpha\beta}(\bar{x})} &= -\frac{1}{2\kappa^2}\delta(x',x'')\delta(x',\bar{x})|g(x')|^{\frac{1}{2}}\left(\left[g^{\mu(\alpha}g^{\beta)\rho}-\frac{1}{2}g^{\alpha\beta}g^{\mu\rho}\right]g^{\nu\sigma}+g^{\mu\rho}g^{\nu(\alpha}g^{\beta)\sigma}\right)\\
&\quad -\frac{1}{2\kappa^2}\delta(x',x'')\delta(x',\bar{x})|g(x')|^{\frac{1}{2}}\left(\left[g^{\mu(\alpha}g^{\beta)\sigma}-\frac{1}{2}g^{\alpha\beta}g^{\mu\sigma}\right]g^{\nu\rho}+g^{\mu\sigma}g^{\nu(\alpha}g^{\beta)\rho}\right)\\
&\quad +\frac{1}{2\kappa^2}\delta(x',x'')\delta(x',\bar{x})|g(x')|^{\frac{1}{2}}\left(\left[g^{\mu(\alpha}g^{\beta)\nu}-\frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\right]g^{\rho\sigma}+g^{\mu\nu}g^{\rho(\alpha}g^{\beta)\sigma}\right)
\end{aligned}$$

which can be rewritten symmetrically

$$\begin{aligned}
\frac{\delta G_{g_{\mu\nu}(x')g_{\rho\sigma}(x'')}}{\delta g_{\alpha\beta}(\bar{x})} &= \frac{1}{2\kappa^2}\delta(x',x'')\delta(x',\bar{x})|g(x')|^{\frac{1}{2}}\Big[-\frac{1}{2}g^{\mu\nu}g^{\rho\sigma}g^{\alpha\beta}+g^{\alpha\beta}g^{\mu(\rho}g^{\sigma)\nu}+g^{\mu\nu}g^{\rho(\alpha}g^{\beta)\sigma}\\
&\quad +g^{\rho\sigma}g^{\mu(\alpha}g^{\beta)\nu}-g^{\mu\rho}g^{\nu(\alpha}g^{\beta)\sigma}-g^{\mu\sigma}g^{\nu(\alpha}g^{\beta)\rho}-g^{\nu\rho}g^{\mu(\alpha}g^{\beta)\sigma}-g^{\nu\sigma}g^{\mu(\alpha}g^{\beta)\rho}\Big]
\end{aligned}$$

We can see that the tensor structure is symmetric under $\mu \leftrightarrow \nu$, $\rho \leftrightarrow \sigma$, $\alpha \leftrightarrow \beta$ and $(\mu\nu) \leftrightarrow (\rho\sigma)$, as expected since we have the symmetric metric $G_{ij} = G_{ji}$. The next two terms in the bracket of eq.(C8) can be directly written down

$$\begin{aligned}
\frac{\delta G_{g_{\mu\nu}(x')g_{\alpha\beta}(\bar{x})}}{\delta g_{\rho\sigma}(x'')} &= \frac{1}{2\kappa^2}\delta(x',\bar{x})\delta(x',x'')|g(x')|^{\frac{1}{2}}\Big[-\frac{1}{2}g^{\mu\nu}g^{\rho\sigma}g^{\alpha\beta}+g^{\rho\sigma}g^{\mu(\alpha}g^{\beta)\nu}+g^{\mu\nu}g^{\alpha(\rho}g^{\sigma)\beta}\\
&\quad +g^{\alpha\beta}g^{\mu(\rho}g^{\sigma)\nu}-g^{\mu\alpha}g^{\nu(\rho}g^{\sigma)\beta}-g^{\mu\beta}g^{\nu(\rho}g^{\sigma)\alpha}-g^{\nu\alpha}g^{\mu(\rho}g^{\sigma)\beta}-g^{\nu\beta}g^{\mu(\rho}g^{\sigma)\alpha}\Big]
\end{aligned}$$

$$\begin{aligned}
\frac{\delta G_{g_{\alpha\beta}(\bar{x})g_{\rho\sigma}(x'')}}{\delta g_{\mu\nu}(x')} &= \frac{1}{2\kappa^2}\delta(\bar{x},x'')\delta(\bar{x},x')|g(\bar{x})|^{\frac{1}{2}}\Big[-\frac{1}{2}g^{\mu\nu}g^{\rho\sigma}g^{\alpha\beta}+g^{\mu\nu}g^{\alpha(\rho}g^{\sigma)\beta}+g^{\rho\sigma}g^{\alpha(\mu}g^{\nu)\beta}\\
&\quad +g^{\alpha\beta}g^{\rho(\mu}g^{\nu)\sigma}-g^{\alpha\rho}g^{\beta(\mu}g^{\nu)\sigma}-g^{\alpha\sigma}g^{\beta(\mu}g^{\nu)\rho}-g^{\beta\rho}g^{\alpha(\mu}g^{\nu)\sigma}-g^{\beta\sigma}g^{\alpha(\mu}g^{\nu)\rho}\Big]
\end{aligned}$$

These three components have a common factor,

$$\left[-\frac{1}{2}g^{\mu\nu}g^{\rho\sigma}g^{\alpha\beta}+g^{\rho\sigma}g^{\mu(\alpha}g^{\beta)\nu}+g^{\mu\nu}g^{\alpha(\rho}g^{\sigma)\beta}+g^{\alpha\beta}g^{\mu(\rho}g^{\sigma)\nu}\right] \tag{C9}$$

which is symmetric under $(\mu\nu) \leftrightarrow (\rho\sigma) \leftrightarrow (\alpha\beta)$. Now eq.(C8) can be calculated as

$$\begin{aligned}
&\Gamma^{g_{\lambda\tau}(x)}_{g_{\mu\nu}(x')g_{\rho\sigma}(x'')} \\
&= \int d^4\bar{x}\frac{1}{2}G^{g_{\lambda\tau}(x)g_{\alpha\beta}(\bar{x})}\left[\frac{\delta G_{g_{\mu\nu}(x')g_{\alpha\beta}(\bar{x})}}{\delta g_{\rho\sigma}(x'')}+\frac{\delta G_{g_{\alpha\beta}(\bar{x})g_{\rho\sigma}(x'')}}{\delta g_{\mu\nu}(x')}-\frac{\delta G_{g_{\mu\nu}(x')g_{\rho\sigma}(x'')}}{\delta g_{\alpha\beta}(\bar{x})}\right] \\
&= \frac{1}{4}\delta(x,x')\delta(x',x'')\left(g_{\lambda(\alpha}g_{\beta)\tau}-\frac{1}{2}g_{\lambda\tau}g_{\alpha\beta}\right)\times \\
&\left(\left[-\frac{1}{2}g^{\mu\nu}g^{\rho\sigma}g^{\alpha\beta}+g^{\rho\sigma}g^{\mu(\alpha}g^{\beta)\nu}+g^{\mu\nu}g^{\alpha(\rho}g^{\sigma)\beta}+g^{\alpha\beta}g^{\mu(\rho}g^{\sigma)\nu}\right]\right. \\
&+\left[-g^{\mu\alpha}g^{\nu(\rho}g^{\sigma)\beta}-g^{\mu\beta}g^{\nu(\rho}g^{\sigma)\alpha}-g^{\nu\alpha}g^{\mu(\rho}g^{\sigma)\beta}-g^{\nu\beta}g^{\mu(\rho}g^{\sigma)\alpha}\right] \\
&+\left[-g^{\alpha\rho}g^{\beta(\mu}g^{\nu)\sigma}-g^{\alpha\sigma}g^{\beta(\mu}g^{\nu)\rho}-g^{\beta\rho}g^{\alpha(\mu}g^{\nu)\sigma}-g^{\beta\sigma}g^{\alpha(\mu}g^{\nu)\rho}\right] \\
&\left.-\left[-g^{\mu\rho}g^{\nu(\alpha}g^{\beta)\sigma}-g^{\mu\sigma}g^{\nu(\alpha}g^{\beta)\rho}-g^{\nu\rho}g^{\mu(\alpha}g^{\beta)\sigma}-g^{\nu\sigma}g^{\mu(\alpha}g^{\beta)\rho}\right]\right)
\end{aligned} \tag{C10}$$

The index contract can be computed directly. We finally have

$$\begin{aligned}
\Gamma^{g_{\lambda\tau}(x)}_{g_{\mu\nu}(x')g_{\rho\sigma}(x'')} &= \delta(x,x')\delta(x',x'')\left(-\frac{1}{8}g^{\mu\nu}g^{\rho\sigma}g_{\lambda\tau}-\delta^{(\mu}_{(\lambda}g^{\nu)(\rho}\delta^{\sigma)}_{\tau)}\right. \\
&\left.+\frac{1}{4}\left[g^{\rho\sigma}\delta^{\mu}_{(\lambda}\delta^{\nu}_{\tau)}+g^{\mu\nu}\delta^{\rho}_{(\lambda}g^{\sigma}_{\tau)}+g_{\lambda\tau}g^{\mu(\rho}g^{\sigma)\nu}\right]\right)
\end{aligned} \tag{C11}$$

We can summarize the non-vanishing Christoffel connection components as follows

$$\begin{aligned}
\Gamma^{g_{\mu\nu}(x)}_{A_\lambda(x')A_\tau(x'')} &= \frac{1}{2}\kappa^2\delta^{(\lambda}_{\mu}\delta^{\tau)}_{\nu}\delta(x,x')\delta(x',x'') \\
\Gamma^{A_\mu(x)}_{A_\nu(x')g_{\alpha\beta}(x'')} &= \frac{1}{4}\left(\delta^{\nu}_{\mu}g^{\alpha\beta}-2\delta^{(\alpha}_{\mu}g^{\beta)\nu}\right)\delta(x,x')\delta(x,x'') \\
\Gamma^{A_\mu(x)}_{g_{\alpha\beta}(x'')A_\nu(x')} &= \Gamma^{A_\mu(x)}_{A_\nu(x')g_{\alpha\beta}(x'')} \\
\Gamma^{g_{\lambda\tau}(x)}_{g_{\mu\nu}(x')g_{\rho\sigma}(x'')} &= \left[-\delta^{(\mu}_{(\lambda}g^{\nu)(\rho}\delta^{\sigma)}_{\tau)}+\frac{1}{4}g^{\mu\nu}\delta^{\rho}_{(\lambda}\delta^{\sigma}_{\tau)}+\frac{1}{4}g^{\rho\sigma}\delta^{\mu}_{(\lambda}\delta^{\nu}_{\tau)}\right. \\
&\left.+\frac{1}{4}\left(g_{\lambda\tau}g^{\mu(\rho}g^{\sigma)\nu}-\frac{1}{2}g_{\lambda\tau}g^{\mu\nu}g^{\rho\sigma}\right)\right]\delta(x,x'')\delta(x',x'')
\end{aligned} \tag{C12}$$

Appendix D: Functional derivatives

Let us now calculate the functional derivatives $S_{,i}$ and $S_{,ij}$. The functional derivatives over the graviton and gauge fields on a general background are decomposed as

$$\frac{\delta S}{\delta g_{\mu\nu}(x)} = \frac{\delta\left(S_G+S_M\right)}{\delta g_{\mu\nu}(x)}=\frac{\delta S_G}{\delta g_{\mu\nu}(x)}+\frac{\delta S_M}{\delta g_{\mu\nu}(x)} \tag{D1}$$

$$\frac{\delta S}{\delta A_\mu(x)} = \frac{\delta\left(S_G+S_M\right)}{\delta A_\mu(x)}=\frac{\delta S_M}{\delta A_\mu(x)} \tag{D2}$$

We shall calculate the above quantities separately.

$$\frac{\delta S_M}{\delta A_\mu(x)} = \frac{1}{4}\int d^4x'|g(x')|^{\frac{1}{2}} g^{\alpha\rho}g^{\beta\sigma}\frac{\delta\left(F_{\rho\sigma}F_{\alpha\beta}\right)}{\delta A_\mu(x)} = \partial_\alpha\left(|g(x)|^{\frac{1}{2}}F^{\mu\alpha}\right) \tag{D3}$$

and

$$\begin{aligned}\frac{\delta S_M}{\delta g_{\mu\nu}(x)} &= \frac{1}{4}\int d^4x' F_{\rho\sigma}F_{\alpha\beta}\frac{\delta\left(|g(x')|^{1/2}g^{\alpha\rho}g^{\beta\sigma}\right)}{\delta g_{\mu\nu}(x)} \\ &= \frac{1}{4}|g(x)|^{\frac{1}{2}}\left[\frac{1}{2}g^{\mu\nu}F^2 - 2F^{\mu}{}_{\sigma}F^{\nu\sigma}\right] = -\frac{1}{2}|g(x)|^{\frac{1}{2}}T^{\mu\nu}\end{aligned} \tag{D4}$$

where we have used $F^2 = F_{\alpha\beta}F^{\alpha\beta}$ for short and defined

$$T^{\mu\nu} = \frac{-2}{\sqrt{g(x)}}\frac{\delta S_M}{\delta g_{\mu\nu}(x)} = F^{\mu}{}_{\sigma}F^{\nu\sigma} - \frac{1}{4}g^{\mu\nu}F^2 \tag{D5}$$

For the functional derivative with respect to the metric, we list the following formulas for convenience.

$$\begin{aligned}\delta R^{\rho}{}_{\sigma\mu\nu} &= \partial_\mu\delta\Gamma^{\rho}_{\nu\sigma} - \partial_\nu\delta\Gamma^{\rho}_{\mu\sigma} + \delta\left[\Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma}\right] \\ \nabla_\lambda\delta\Gamma^{\rho}_{\nu\mu} &= \partial_\lambda\delta\Gamma^{\rho}_{\nu\mu} + \Gamma^{\rho}_{\sigma\lambda}\delta\Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\nu\lambda}\delta\Gamma^{\rho}_{\sigma\mu} - \Gamma^{\sigma}_{\mu\lambda}\delta\Gamma^{\rho}_{\nu\sigma} \\ \delta R^{\rho}{}_{\sigma\mu\nu} &= \nabla_\mu\delta\Gamma^{\rho}_{\nu\sigma} - \nabla_\nu\delta\Gamma^{\rho}_{\mu\sigma} \\ \delta\Gamma^{\rho}_{\mu\nu} &= \frac{1}{2}g^{\rho\sigma}\left[(\delta g_{\mu\sigma})_{;\nu} + (\delta g_{\nu\sigma})_{;\mu} - (\delta g_{\mu\nu})_{;\rho}\right]\end{aligned} \tag{D6}$$

and

$$\begin{aligned}\delta R_{\mu\nu} &= \delta R^{\rho}{}_{\mu\rho\nu} = \nabla_\rho\delta\Gamma^{\rho}_{\nu\mu} - \nabla_\nu\delta\Gamma^{\rho}_{\rho\mu} \\ \delta R &= R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu} = R_{\mu\nu}\delta g^{\mu\nu} + \nabla_\sigma\left[g^{\mu\nu}\delta\Gamma^{\sigma}_{\nu\mu} - g^{\mu\sigma}\delta\Gamma^{\rho}_{\rho\mu}\right] \\ &= R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}g^{\rho\sigma}\left[\delta g_{\rho\sigma;\mu\nu} + \delta g_{\rho\mu;\rho\nu}\right]\end{aligned}$$

Similarly, we have

$$\frac{\delta S_G}{\delta g_{\mu\nu}(x)} = = -\frac{2}{\kappa^2}\int d^4x'\left[\frac{\delta\left(|g(x')|^{\frac{1}{2}}(R-2\Lambda)\right)}{\delta g_{\mu\nu}(x)}\right] = -\frac{2}{\kappa^2}|g(x)|^{\frac{1}{2}}E^{\mu\nu}$$

where we have used

$$\int d^4x'|g(x')|^{\frac{1}{2}}\left(g^{\alpha\beta}\frac{\delta R_{\alpha\beta}}{\delta g_{\mu\nu}(x)}\right) = \text{Surface terms} \tag{D7}$$

and defined $E^{\mu\nu} = \frac{1}{2}(R-2\Lambda)g^{\mu\nu} - R^{\mu\nu}$. The Einstein equation can be obtained by imposing $\frac{\delta S}{\delta g_{\mu\nu}(x)} = 0$,

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu} \tag{D8}$$

In the present paper, we are working in a flat background space-time. Since in this case we expand at a background that doesn't satisfy Einstein equation, we actually deal with the off-shell effective action. The connection term in eq.(B32) is necessary to be included for a gauge condition independent result. For $S_{,i}$, we summarize the final result as

$$\frac{\delta S_M}{\delta g_{\mu\nu}(x)} = \frac{1}{4}|g(x)|^{\frac{1}{2}}\left[\frac{1}{2}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} - 2F^{\mu}{}_{\sigma}F^{\nu\sigma}\right] \tag{D9}$$

$$\frac{\delta S_G}{\delta g_{\mu\nu}(x)} = -\frac{2}{\kappa^2}|g(x)|^{\frac{1}{2}}\left[\frac{1}{2}(R-2\Lambda)g^{\mu\nu} - R^{\mu\nu}\right] \tag{D10}$$

$$\frac{\delta S_M}{\delta A_\mu(x)} = \partial_\alpha\left(|g(x)|^{\frac{1}{2}}F^{\mu\alpha}\right), \ \frac{\delta S_G}{\delta A_\mu(x)} = 0 \tag{D11}$$

The above formulas are true for general background space-time $\bar{g}_{\mu\nu}$.

Although we can expand the Lagrangian eq.(B33) straightforwardly for a flat background space-time, here we shall calculate $S_{,ij}$ for the expansion. We may use the following equations for convenience,

$$\frac{\delta\left(|g(x')|^{\frac{1}{2}}F^2\right)}{\delta g_{\mu\nu}(x)} = |g(x)|^{\frac{1}{2}}\delta(x',x)\left[\frac{1}{2}g^{\mu\nu}F^2 - 2F^{\mu}{}_{\sigma}F^{\nu\sigma}\right]$$

$$\frac{\delta\left(|g(x')|^{\frac{1}{2}}F^{\alpha\beta}\right)}{\delta g_{\mu\nu}(x)} = \frac{\delta\left(|g(x')|^{\frac{1}{2}}g^{\alpha\rho}g^{\beta\sigma}F_{\rho\sigma}\right)}{\delta g_{\mu\nu}(x)}$$

$$= |g(x')|^{\frac{1}{2}}\delta(x',x)\left[\frac{1}{2}g^{\mu\nu}F^{\alpha\beta} - g^{\alpha(\mu}F^{\nu)\beta} + g^{\beta(\mu}F^{\nu)\alpha}\right]$$

In computing $S_{,ij}$, the following components are straightforward,

$$\frac{\delta^2 S_G}{\delta A_\mu(x)\delta A_\nu(x')} = 0 \tag{D12}$$

$$\begin{aligned}\frac{\delta^2 S_M}{\delta A_\mu(x)\delta A_\nu(x')} &= \partial_\alpha\left(|g(x)|^{\frac{1}{2}}\frac{\delta F^{\mu\alpha}}{\delta A_\nu(x')}\right)\\ &= \partial_\alpha\left(|g(x)|^{\frac{1}{2}}\left[\partial^\mu\delta^{\alpha\nu} - \partial^\alpha\delta^{\mu\nu}\right]\delta(x,x')\right)\end{aligned} \tag{D13}$$

$$\begin{aligned}\frac{\delta^2 S_M}{\delta A_\mu(x)\delta g_{\alpha\beta}(x')} &= \partial_\nu\left(\frac{\delta\left[|g(x)|^{\frac{1}{2}}F^{\mu\nu}\right]}{\delta g_{\alpha\beta}(x')}\right)\\ &= \partial_\nu\left(g(x)|^{\frac{1}{2}}\delta(x,x')\left[\frac{1}{2}g^{\alpha\beta}F^{\mu\nu} - g^{\mu(\alpha}F^{\beta)\nu} + g^{\nu(\alpha}F^{\beta)\mu}\right]\right)\end{aligned} \tag{D14}$$

The rest parts are much more complicated. The matter part has the form

$$\frac{\delta^2 S_M}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}(x')} = \frac{1}{4}\frac{\delta}{\delta g_{\alpha\beta}(x')}\left(|g(x)|^{\frac{1}{2}}\left[\frac{1}{2}g^{\mu\nu}F^2 - 2F^\mu{}_\sigma F^{\nu\sigma}\right]\right)$$

$$= |g(x)|^{\frac{1}{2}}\delta(x,x')\Bigg(\frac{1}{16}g^{\mu\nu}g^{\alpha\beta}F^2 - \frac{1}{8}F^2 g^{\mu(\alpha}g^{\beta)\nu} + \frac{1}{2}F^\nu{}_\sigma g^{\mu(\alpha}F^{\beta)\sigma}$$

$$+\frac{1}{2}F^\mu{}_\sigma g^{\nu(\alpha}F^{\beta)\sigma} - \frac{1}{2}F^\mu{}_\sigma g^{\sigma(\alpha}F^{\beta)\nu} - \frac{1}{4}g^{\mu\nu}F^\alpha{}_\sigma F^{\beta\sigma} - \frac{1}{4}g^{\alpha\beta}F^\mu{}_\sigma F^{\nu\sigma}\Bigg)$$

For the gravity part, we have

$$\frac{\delta^2 S_G}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}(x')}$$

$$= -\frac{1}{\kappa^2}|g(x)|^{\frac{1}{2}}\delta(x,x')\Bigg((R-2\Lambda)\left[\frac{1}{2}g^{\mu\nu}g^{\alpha\beta} - g^{\mu(\alpha}g^{\beta)\nu}\right] - g^{\mu\nu}R^{\alpha\beta} - g^{\alpha\beta}R^{\mu\nu}$$

$$+2\left[g^{\alpha(\mu}R^{\beta)\nu} + g^{\alpha(\nu}R^{\beta)\mu}\right]\Bigg) - \frac{1}{\kappa^2}|g(x)|^{\frac{1}{2}}\left[g^{\mu\nu}g^{\rho\sigma} - 2g^{\mu\rho}g^{\sigma\nu}\right]\frac{\delta R_{\rho\sigma}}{\delta g_{\alpha\beta}(x')}$$

where $\frac{\delta R_{\rho\sigma}}{\delta g_{\alpha\beta}(x')}$ can be worked out by using eq.(D6).

Appendix E: Euclidean flat Background

In the following discussion, we will focus on the flat background space-time and consider the one-loop contribution to the gauge effective action from the graviton. We expand the fields, $\varphi^i = (g_{\mu\nu}, A_\mu)$, at the background-fields, $\bar{\varphi}^i = (\delta_{\mu\nu}, \bar{A}_\mu)$,

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu}; \quad A_\mu = \bar{A}_\mu + a_\mu \tag{E1}$$

Expansion of the action in flat background space-time is straightforward by directly replacing the fields with above equations. One can also work out first the functional derivatives, $S_{,i}$ and $S_{,ij}$, and then consider the effective Lagrangian $\frac{1}{2}\eta^i\eta^j\left[S_{,ij} - \Gamma^k_{ij}S_{,k} + \frac{1}{2\Omega}\chi_\alpha{}^{,i}\chi^\alpha{}_{,j}\right]$, as we shall show below in detail.

Using the formulas given in the previous appendix sections and imposing the flat background space-time, we have

$$\left.\frac{\delta S_M}{\delta g_{\mu\nu}(x)}\right|_{\bar{\varphi}^i} = \frac{1}{4}\left[\frac{1}{2}\delta^{\mu\nu}\bar{F}^2 - 2\bar{F}^\mu{}_\sigma\bar{F}^{\nu\sigma}\right], \ \left.\frac{\delta S_G}{\delta g_{\mu\nu}(x)}\right|_{\bar{\varphi}^i} = \frac{2}{\kappa^2}\Lambda\delta^{\rho\sigma} \tag{E2}$$

$$\left.\frac{\delta S_M}{\delta A_\mu(x)}\right|_{\bar{\varphi}^i} = \partial_\alpha\bar{F}^{\mu\alpha}, \ \left.\frac{\delta S_G}{\delta A_\mu(x)}\right|_{\bar{\varphi}^i} = 0 \tag{E3}$$

For $S_{,ij}$, the most complicated one is $\frac{\delta^2 S}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}(x')}$, and we need the following result

$$\left.\frac{\delta R_{\mu\nu}}{\delta g_{\alpha\beta}(x')}\right|_{\bar{\varphi}^i} = \frac{1}{2}\left[2\delta^{(\alpha}_{(\mu}\partial^{\beta)}\partial_{\nu)} - \delta^{(\alpha}_{\mu}\delta^{\beta)}_{\nu}\Box - \delta^{\alpha\beta}\partial_\mu\partial_\nu\right]\delta(x,x')$$

putting all together, we can show that

$$\begin{aligned}
&\left.\frac{\delta^2 S_G}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}(x')}\right|_{\bar{\varphi}^i} \\
=\ & \frac{1}{\kappa^2}\delta(x,x')\Big[\partial^{(\mu}\delta^{\nu)(\alpha}\partial^{\beta)} + \left(\delta^{\alpha\beta}\delta^{\mu\nu} - \delta^{\mu(\alpha}\delta^{\beta)\nu}\right)\Box - \delta^{\mu\nu}\partial^\alpha\partial^\beta - \delta^{\alpha\beta}\partial^\mu\partial^\nu\Big] \\
& + \frac{2\Lambda}{\kappa^2}\delta(x,x')\left[\frac{1}{2}\delta^{\mu\nu}\delta^{\alpha\beta} - \delta^{\mu(\alpha}\delta^{\beta)\nu}\right]
\end{aligned}$$

We may summarize the following formulas with flat background space-time,

$$\begin{aligned}
\left.\frac{\delta^2 S_G}{\delta A_\mu(x)\delta A_\nu(x')}\right|_{\bar{\varphi}^i} &= \left.\frac{\delta^2 S_G}{\delta A_\mu(x)\delta g_{\alpha\beta}(x')}\right|_{\bar{\varphi}^i} = 0 \\
\left.\frac{\delta^2 S_M}{\delta A_\mu(x)\delta A_\nu(x')}\right|_{\bar{\varphi}^i} &= \left[\partial^\mu\partial^\nu - \partial^2\delta^{\mu\nu}\right]\delta(x,x') \\
\left.\frac{\delta^2 S_M}{\delta A_\mu(x)\delta g_{\alpha\beta}(x')}\right|_{\bar{\varphi}^i} &= \partial_\nu\left(\delta(x,x')\left[\frac{1}{2}\delta^{\alpha\beta}\bar{F}^{\mu\nu} - \delta^{\mu(\alpha}\bar{F}^{\beta)\nu} + \delta^{\nu(\alpha}\bar{F}^{\beta)\mu}\right]\right)
\end{aligned}$$

and

$$\begin{aligned}
\left.\frac{\delta^2 S_M}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}(x')}\right|_{\bar{\varphi}^i} &= \delta(x,x')\Bigg(\left[\frac{1}{16}\delta^{\mu\nu}\delta^{\alpha\beta}\bar{F}^2 - \frac{1}{8}\bar{F}^2\delta^{\mu(\alpha}\delta^{\beta)\nu}\right] \\
&\quad + \frac{1}{2}\bar{F}^\nu{}_\sigma\delta^{\mu(\alpha}\bar{F}^{\beta)\sigma} + \frac{1}{2}\bar{F}^\mu{}_\sigma\delta^{\nu(\alpha}\bar{F}^{\beta)\sigma} - \frac{1}{2}\bar{F}^\mu{}_\sigma\delta^{\sigma(\alpha}\bar{F}^{\beta)\nu} \\
&\quad - \frac{1}{4}\delta^{\mu\nu}\bar{F}^\alpha{}_\sigma\bar{F}^{\beta\sigma} - \frac{1}{4}\delta^{\alpha\beta}\bar{F}^\mu{}_\sigma\bar{F}^{\nu\sigma}\Bigg) \\
\left.\frac{\delta^2 S_G}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}(x')}\right|_{\bar{\varphi}^i} &= \frac{2\Lambda}{\kappa^2}\delta(x,x')\left[\frac{1}{2}\delta^{\mu\nu}\delta^{\alpha\beta} - \delta^{\mu(\alpha}\delta^{\beta)\nu}\right] + \frac{1}{\kappa^2}\delta(x,x')\times \\
&\quad \left[\partial^{(\mu}\delta^{\nu)(\alpha}\partial^{\beta)} + \left(\delta^{\alpha\beta}\delta^{\mu\nu} - \delta^{\mu(\alpha}\delta^{\beta)\nu}\right)\Box - \delta^{\mu\nu}\partial^\alpha\partial^\beta - \delta^{\alpha\beta}\partial^\mu\partial^\nu\right]
\end{aligned}$$

so far we have the pieces to calculate the covariant derivative for the classical action with respect to φ^i and to expand the terms which are necessary for one-loop calculation of Vilkovisky-DeWitt effective action. We shall work out the needed effective action by expanding and truncating piece by piece.

1. Ordinary derivative terms $\frac{1}{2}\eta^i S_{,ij}\eta^j$

Let us first consider the quadratic terms on the quantum gauge field a_μ,

$$\begin{aligned}\frac{1}{2}aS_{,AA}a &= \frac{1}{2}\int d^4xd^4x' a_\mu(x)\left.\frac{\delta^2 S_M}{\delta A_\mu(x)\delta A_\nu(x')}\right|_{\bar\varphi^i} a_\nu(x') \\ &= \frac{1}{2}\int d^4x a_\mu(x)\left[\partial^\mu\partial^\nu - \partial^2\delta^{\mu\nu}\right]a_\nu(x)\end{aligned} \quad \text{(E4)}$$

This part together with the gauge fixing term will give the propagator of gauge boson. With including cosmological constant, it will be seen in the connection terms that other terms will contribution as well. The quadratic terms cross on graviton $h_{\mu\nu}$ and gauge field a_μ are given by

$$\begin{aligned}\frac{1}{2}aS_{,Ag}\kappa h &= \frac{1}{2}\int d^4xd^4x' a_\mu(x)\left.\frac{\delta^2 S_M}{\delta A_\mu(x)\delta g_{\alpha\beta}(x')}\right|_{\bar\varphi^i}\kappa h_{\alpha\beta}(x') \\ &= \frac{\kappa}{2}\int d^4x\left[\frac{1}{2}h\bar F^{\mu\nu}\partial_\mu a_\nu - \bar F_\beta{}^\mu h^{\nu\beta}\partial_\nu a_\mu + \bar F_\beta{}^\nu h^{\alpha\beta}\partial_\nu a_\alpha\right]\end{aligned} \quad \text{(E5)}$$

and the same terms for $\frac{1}{2}\kappa hS_{,gA}a$. There are also quadratic terms on graviton field $h_{\mu\nu}$,

$$\begin{aligned}\frac{1}{2}\kappa hS_{,gg}\kappa h &= \frac{\kappa^2}{2}\int d^4xd^4x' h_{\mu\nu}(x)\left.\frac{\delta^2 S_G}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}(x')}\right|_{\bar\varphi^i} h_{\alpha\beta}(x') \\ &= \int d^4x\left(\Lambda\left[\frac{1}{2}h^2 - h^{\mu\nu}h_{\mu\nu}\right] + \left[\frac{1}{4}h\partial^2 h - \frac{1}{2}h^{\mu\nu}\partial^2 h_{\mu\nu} - \left(\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h\right)^2\right]\right)\end{aligned} \quad \text{(E6)}$$

The first term associated with Λ in the parenthesis may act as a mass term for graviton, and will display itself in the graviton propagator.

2. Manifold connection terms $-\frac{1}{2}\eta^i\Gamma^k_{ij}S_{,k}\eta^j$

Since we are working in a flat background space-time, which is not a solution of Einstein equation in the presence of matter fields, we need to include the connection terms for yielding a gauge fixing condition independent result in the Vilkovisky-DeWitt's framework. The quadratic terms on quantum gauge field a_μ are,

$$\begin{aligned}-\frac{1}{2}a\Gamma^g_{AA}aS_{,g} &= -\frac{1}{2}\int d^4xd^4x'd^4x'' a_\lambda(x')\Gamma^{g_{\mu\nu}(x)}_{A_\lambda(x')A_\tau(x'')}a_\tau(x'')\left.\frac{\delta S}{\delta g_{\mu\nu}(x)}\right|_{\bar\varphi^i} \\ &= \int d^4x\left(-\frac{1}{2}\Lambda\delta^{\mu\nu} - \frac{1}{8}\kappa^2\left[\frac{1}{4}\delta^{\mu\nu}\bar F^2 - \bar F^\mu{}_\sigma\bar F^{\nu\sigma}\right]\right)a_\mu a_\nu\end{aligned} \quad \text{(E7)}$$

As we can see from the above equation that, with the non-zero cosmological constant, the connection induced interactions with the term $2\Lambda a_\mu a^\nu$ will change the gauge propagator. This also happens in the graviton part as we can see above. The quadratic terms cross on graviton $h_{\mu\nu}$ and gauge field a_μ are,

$$
\begin{aligned}
-\frac{1}{2}a\Gamma^A_{Ag}\kappa h S_{,A} &= -\frac{1}{2}\int d^4x d^4x' d^4x'' a_\nu(x')\Gamma^{A_\mu(x)}_{A_\nu(x')g_{\alpha\beta}(x'')}\kappa h_{\alpha\beta}(x'')\left.\frac{\delta S}{\delta A_\mu(x)}\right|_{\bar{\varphi}^i} \\
&= -\frac{\kappa}{8}\int d^4x\left[\delta^{\alpha\beta}\delta^\nu_\mu - 2\delta^{\nu(\alpha}\delta^{\beta)}_\mu\right]\partial_\lambda \bar{F}^{\mu\lambda}h_{\alpha\beta}a_\nu
\end{aligned} \tag{E8}
$$

which involves the term $\partial_\lambda \bar{F}^{\mu\lambda}$ and will not contribute the corrections to F^2 operator in our calculation. The quadratic terms on graviton $h_{\mu\nu}$ are given by,

$$
\begin{aligned}
-\frac{1}{2}\kappa h\Gamma^g_{gg}\kappa h S_{,g} &= -\frac{\kappa^2}{2}\int d^4x d^4x' d^4x'' h_{\mu\nu}(x')\Gamma^{g_{\lambda\tau}(x)}_{g_{\mu\nu}(x')g_{\rho\sigma}(x'')}h_{\rho\sigma}(x'')\left.\frac{\delta S}{\delta g_{\lambda\tau}(x)}\right|_{\bar{\varphi}^i} \\
&= \frac{\kappa^2}{4}\int d^4x h_{\mu\nu}h_{\rho\sigma}\left[\frac{1}{4}\delta^{\mu\rho}\delta^{\sigma\nu}\bar{F}^2 - \frac{1}{8}\delta^{\mu\nu}\delta^{\rho\sigma}\bar{F}^2 + \frac{1}{2}\delta^{\rho\sigma}\bar{F}^\mu{}_\alpha\bar{F}^{\nu\alpha} - \delta^{\nu\rho}\bar{F}^\mu{}_\alpha\bar{F}^{\sigma\alpha}\right]
\end{aligned} \tag{E9}
$$

These are interacting terms between graviton and gauge boson from the connection terms, which is crucial for gauge condition independent calculation.

3. Gauge Fixing terms $\frac{1}{4\Omega}\eta^i K_{\alpha i}K^\alpha_j\eta^j$

For getting proper propagators, we shall also include the gauge fixing term. The Landau-DeWitt gauge is defined by eq. (B30) with $\chi_\alpha = K^i_\alpha[\bar{\varphi}]g_{ij}[\bar{\varphi}]\eta^j = 0$. For the gravity, we have

$$
\begin{aligned}
\chi_\alpha &= \int d^4x' d^4x'' \Bigg(\left[-\delta_{\mu\alpha}\partial'_\nu - \delta_{\alpha\nu}\partial'_\mu\right]\delta(x',x)\frac{1}{\kappa^2}\left(\delta^{\mu(\rho}\delta^{\sigma)\nu} - \frac{1}{2}\delta^{\mu\nu}\delta^{\rho\sigma}\right)\delta(x',x'')\kappa h_{\rho\sigma}(x'') \\
&\quad + \left[-\bar{A}_{\mu,\alpha'}(x') - \bar{A}_\alpha(x')\partial'_\mu\right]\delta(x',x)\delta^{\mu\nu}\delta(x',x'')a_\nu(x'')\Bigg) \\
&= \frac{2}{\kappa}\left[\partial^\mu h_{\mu\alpha} - \frac{1}{2}\partial_\alpha h\right] + \left[a^\mu\bar{F}_{\mu\alpha} + \bar{A}_\alpha\partial^\mu a_\mu\right]
\end{aligned} \tag{E10}
$$

with $h = \delta_{\mu\nu}h^{\mu\nu}$. For the gauge field, we yield

$$
\chi = \int d^4x' d^4x''\left[\partial'_\mu\delta(x',x)\right]\delta^{\mu\nu}\delta(x',x'')a_\nu(x'') = -\partial^\mu a_\mu \tag{E11}
$$

Thus the Landau-DeWitt gauge conditions ($\omega = 1$) are found to be

$$
\chi_\lambda = \frac{2}{\kappa}(\partial^\mu h_{\mu\lambda} - \frac{1}{2}\partial_\lambda h) + \omega(\bar{A}_\lambda\partial^\mu a_\mu + a^\mu\bar{F}_{\mu\lambda}) \tag{E12}
$$

$$
\chi = -\partial^\mu a_\mu \tag{E13}
$$

where ω is a parameter introduced [25] for a comparison with the traditional background-field method with harmonic gauge($\omega = 0$). It is tempting to impose $\partial^\mu a_\mu = 0$ in eq.(E12), we shall discuss it late on as this term can bring quartic divergences which may break the $U(1)$ gauge invariance.

The gauge fixing term can be written explicitly as

$$S_{GF} = \frac{1}{4\Omega}\eta^i K_{\alpha i} K^\alpha_j \eta^j = \frac{1}{4\xi}(\chi_\lambda)^2 + \frac{1}{4\zeta}(\chi)^2 \tag{E14}$$

where ξ and ζ are gauge fixing parameters for gravity and gauge fields, respectively. The gauge fixing terms are given by

$$\begin{aligned} S_{GF} &= \frac{1}{\kappa^2\xi}\left[\partial^\mu h_{\mu\lambda} - \frac{1}{2}\partial_\lambda h\right]^2 + \frac{1}{4\zeta}\left[\partial^\mu a_\mu\right]^2 + \frac{\omega^2}{4\xi}\left[\bar{A}_\lambda \partial^\mu a_\mu + a^\mu \bar{F}_{\mu\lambda}\right]^2 \\ &+ \frac{\omega}{\kappa\xi}\left[\partial^\mu h_{\mu\lambda} - \frac{1}{2}\partial_\lambda h\right]\left[\bar{A}_\lambda \partial^\mu a_\mu + a^\mu \bar{F}_{\mu\lambda}\right] \end{aligned} \tag{E15}$$

4. Ghost part

It has to include the ghost's contributions as we are working in a gauge that induces the ghost-gauge coupling. The action of the ghost part is

$$\begin{aligned} S_{GH} &= \bar{c}_\alpha Q^\alpha{}_\beta[\bar{\varphi}] c^\beta = \bar{c}^\lambda \frac{\delta\chi_\lambda}{\delta\varphi^i}\frac{\delta\varphi^i}{\delta\epsilon^\rho}\bigg|_{\bar{\varphi}^i} c^\rho + \bar{c}^\lambda \frac{\delta\chi_\lambda}{\delta\varphi^i}\frac{\delta\varphi^i}{\delta\epsilon}\bigg|_{\bar{\varphi}^i} c + \bar{c}\frac{\delta\chi}{\delta\varphi^i}\frac{\delta\varphi^i}{\delta\epsilon}\bigg|_{\bar{\varphi}^i} c + \bar{c}\frac{\delta\chi}{\delta\varphi^i}\frac{\delta\varphi^i}{\delta\epsilon^\rho}\bigg|_{\bar{\varphi}^i} c^\rho \\ &= S_{GH0} + S_{GH1} + S_{GH2} \end{aligned}$$

where c^ρ and c denotes the corresponding ghost for gravity and gauge feilds, respectively. For the evaluation of one-loop two-point Green's function, we can drop terms with quantum fields $h_{\mu\nu}$ and a_μ since the ghost and its anti-ghost will form a loop, one more quantum field needs contract with another quantum field, forming the second loop. The action is the sum of the following four parts,

$$\begin{aligned} \bar{c}\frac{\delta\chi}{\delta\varphi^i}\frac{\delta\varphi^i}{\delta\epsilon}\bigg|_{\bar{\varphi}^i} c &= \int d^4x d^4x' d^4\bar{x}\left[\bar{c}(x)\frac{\delta\chi(x)}{\delta\varphi^i(x')}K^{\varphi^i}(x',\bar{x})c(\bar{x})\right] \\ = \int d^4x\left[-\bar{c}(x)\right] d^4x'\left[\partial^\mu\delta(x,x')\partial'_\mu c(x')\right] &= \int d^4x\left[-\bar{c}\Box c\right] \end{aligned}$$

which is the free part of Lagrangian for the ghost of the $U(1)$ gauge theory. And

$$\begin{aligned} \bar{c}^\lambda\frac{\delta\chi_\lambda}{\delta\varphi^i}\frac{\delta\varphi^i}{\delta\epsilon^\rho}\bigg|_{\bar{\varphi}^i} c^\rho &= \int d^4x d^4x' d^4\bar{x}\left[\bar{c}^\lambda(x)\frac{\delta\chi_\lambda(x)}{\delta\varphi^i(x')}K^{\bar{\varphi}^i}{}_\rho(x',\bar{x})c^\rho(\bar{x})\right] \\ = \int d^4x\left[-\frac{2}{\kappa^2}\bar{c}^\lambda\Box c_\rho - \omega\bar{c}^\lambda\bar{F}^\nu{}_\lambda\bar{A}_{\nu,\rho}c^\rho - \omega\bar{c}^\lambda\bar{F}^\nu{}_\lambda\bar{A}_\rho\partial_\nu c^\rho - \omega\bar{c}^\lambda\bar{A}_\lambda\bar{A}_\rho\Box c^\rho - \omega\bar{c}^\lambda\bar{A}_\lambda\bar{A}_{\nu,\rho}\partial^\nu c^\rho\right] \end{aligned}$$

where the first term in the bracket gives the free part of Lagrangian for the gravitational ghost introduced by fixing the general coordinate transformation, and other terms are treated as interactions. These interactions are proportional to ω, and indicate that in harmonic gauge $\omega = 0$, gravitational ghost and anti-ghost do not interact with gauge fields. However, there are still interactions like $\bar{c}Ac^{\mu}$ in the Lagrangian for arbitrary ω.

$$\bar{c}^{\lambda}\frac{\delta\chi_{\lambda}}{\delta\varphi^{i}}\frac{\delta\varphi^{i}}{\delta\epsilon}\bigg|_{\bar{\varphi}^{i}} c = \int d^4x d^4x' d^4\bar{x}\left[\bar{c}^{\lambda}(x)\frac{\delta\chi_{\lambda}(x)}{\delta\varphi^{i}(x')}K^{\bar{\varphi}^{i}}(x',\bar{x})c(\bar{x})\right]$$
$$= \omega\int d^4x\left[\bar{c}^{\lambda}\bar{A}_{\lambda}\Box c + \bar{c}^{\lambda}\bar{F}^{\mu}_{\ \lambda}\partial_{\mu}c\right],$$

and

$$\bar{c}\frac{\delta\chi}{\delta\varphi^{i}}\frac{\delta\varphi^{i}}{\delta\epsilon^{\rho}}\bigg|_{\bar{\varphi}^{i}} c^{\rho} = \int d^4x d^4x' d^4\bar{x}\left[\bar{c}(x)\frac{\delta\chi(x)}{\delta\varphi^{i}(x')}K^{\bar{\varphi}^{i}}{}_{\rho}(x',\bar{x})c^{\rho}(\bar{x})\right]$$
$$= \int d^4x\left[\bar{c}\bar{A}^{\mu}_{\ ,\mu\rho}c^{\rho} + \bar{c}\bar{A}_{\mu,\rho}c^{\rho,\mu} + \bar{c}\bar{A}_{\rho}\Box c^{\rho} + \bar{c}\bar{A}_{\rho,\mu}c^{\rho,\mu}\right].$$

[1] G. ‘t Hooft and M. Veltman, Ann. Inst. Henri Poincar A **20**, 69 (1974).

[2] S. Deser and P. van Nieuwenhuizen, Phys. Rev. Lett. **32**, 245, Phys. Rev. D **10**, 401, Phys. Rev. D **10**, 410 (1974).

[3] S. Deser, H. Tsao, and P. van Nieuwenhuizen, Phys. Rev. D **10**, 3337 (1974).

[4] J. F. Donoghue, Phys. Rev. Lett. **72**, 2996 (1994), Phys. Rev. D **50**, 3874 (1994).

[5] Cliff P. Burgess, Living Rev. Relativity, 7, 5(2004).

[6] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. **96**, 231601 (2006).

[7] A. R. Pietrykowski, Phys. Rev. Lett. **98**, 061801 (2007).

[8] D. J. Toms, Phys. Rev. D **76**, 045015 (2007).

[9] G. A. Vilkovisky, Nucl. Phys. B **234**, 125 (1984), *The Quantum Theory of Gravity*, edited by S. M. Christensen (Adam Hilger, Bristol, 1984).

[10] B. S. DeWitt, in *Quantum Field Theory and Quantum Statistics, Volume 1*, edited by I. A. Batalin, C. J. Isham, and G. A. Vilkovisky (Adam Hilger, Bristol, 1987).

[11] G. ‘t Hooft and M. Veltman, Nucl. Phys. B **44**, 189 (1972).

[12] D. Ebert, J. Plefka and A. Rodigast, Phys. Lett. **B**660, 579(2008).

[13] Y. Tang and Y. L. Wu, Comm. Theo. Phys. **54**, 1040(2010), arXiv:0807.0331v2 [hep-ph].

[14] Y. L. Wu, Int. J. Mod. Phys. **A**18, 5363 (2003); Mod. Phys. Lett. **A**19, 2191 (2004).

[15] J. E. Daum, U. Harst and M. Reuter, JHEP 1001,084(2010).

[16] Feng Wu and Ming Zhong, Phys. Lett. B659, 694(2008), Phys. Rev. D 78, 085010 (2008).

[17] A. Rodigast and T. Schuster, Phys. Rev. D 79, 125017(2009), Phys. Rev. Lett. 104, 081301 (2010).

[18] O. Zanusso, L. Zambelli, G.P. Vacca and R. Percacci, Phys.Lett. B689, 90(1010).

[19] Paul T. Mackay and David J. Toms, Phys. Lett. B684, 251(2010).

[20] M. M. Anber, J. F. Donoghue and M. El-Houssieny, Phys. Rev. **D83**, 124003 (2011). [arXiv:1011.3229 [hep-th]].

[21] Y. Tang, Y. -L. Wu, [arXiv:1012.0626 [hep-ph]].

[22] E. Gerwick, Eur. Phys. J. **C71**, 1676 (2011). [arXiv:1012.1118 [hep-ph]].

[23] John Ellis and Nick E. Mavromatos, arXiv:1012:4353 [hep-th].

[24] S. Folkerts, D. F. Litim, J. M. Pawlowski, [arXiv:1101.5552 [hep-th]], D. F. Litim,

[arXiv:1102.4624 [hep-th]].

[25] D. J. Toms, Phys. Rev. Lett. **101**, 131301 (2008), Phys. Rev. D **80**, 064040(2009).

[26] D. J. Toms, Nature 468, 56 (2010).

[27] H. -J. He, X. -F. Wang, Z. -Z. Xianyu, Phys. Rev. **D83**, 125014 (2011).

[28] A. A. Slavnov, Theor. Math. Phys. **10**, 99-107 (1972), J. C. Taylor, Nucl. Phys. **B33**, 436-444 (1971).

[29] Steven Weinberg, The Quantum Theory of fields, Volume I and II (Cambridge University Press, Cambridge, 2005).

[30] Y. -B. Dai, Y. -L. Wu, Eur. Phys. J. **C39**, S1-S8 (2005). [hep-ph/0304075].

[31] Y. -L. Ma, Y. -L. Wu, Int. J. Mod. Phys. **A21**, 6383-6456 (2006),[hep-ph/0509083];
Y. -L. Ma, Y. -L. Wu, Phys. Lett. **B647**, 427-430 (2007). [hep-ph/0611199].

[32] J. -W. Cui, Y. -L. Wu, Int. J. Mod. Phys. **A23**, 2861-2913 (2008). [arXiv:0801.2199 [hep-ph]],
J. -W. Cui, Y. Tang, Y. -L. Wu, Phys. Rev. **D79**, 125008 (2009), [arXiv:0812.0892 [hep-ph]],
J. -W. Cui, Y. -L. Ma, Y. -L. Wu, Phys. Rev. **D84**, 025020 (2011). [arXiv:1103.2026 [hep-ph]].

[33] D. Huang, Y. -L. Wu, [arXiv:1108.3603 [hep-ph]].

[34] L. Parker and D. J. Toms, *Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity* (Cambridge University Press, Cambridge, 2009).

[35] B. S. DeWitt, *The Dynamical Theory of Groups and Fields* (Gordon and Breach, New York, 1965).

[36] L. D. Faddeev and V. N. Popov, Phys. Lett. B **25**, 29 (1967).

[37] E. S. Fradkin and A. A. Tseytlin, Nucl. Phys. B **234**, 509 (1984).

[38] A. O. Barvinsky and G. A. Vilkovisky, Phys. Reports **119**, 1 (1985).

[39] S. D. Odintsov, Phys. Lett. B262, 394(1991)

[40] S. R. Huggins, G. Kunstatter, H. P. Leivo, and D. J. Toms, Nucl. Phys. B **301**, 627 (1987).

[41] R. Mertig, M. Boehm, and A. Denner, Comput. Phys. Commun. **64**, 345(1991).

[42] N. K. Nielsen, arXiv:1109.2699 [hep-th].